6/13/05

Activity 6 Solutions: Entropy and The Laws of Thermodynamics

6.1 Order, Disorder, and Entropy

1) Ordered and Disordered Systems

a) Deal three cards from the deck on your table. Do the three cards have the same picture? If not, return the cards to the deck, shuffle, and deal three cards again. Repeat until you have dealt three cards with the same picture (an ordered set). How many disordered sets did you deal before dealing an ordered set? ________

b) If you continued dealing sets of three cards, which type of set would you expect to deal more frequently: ordered sets or disordered sets?

disordered sets

c) How can you relate your results with the cards to ordered and disordered systems in nature? Which would you think are more common – ordered systems or disordered systems? __**disordered systems**_

2) Ordered and Disordered Checkers

In the next activity, we use checkers on a four-square board to help explain why disordered systems are more common than ordered systems. To do this, we find the probability of drawing at random checkers whose colors match the colors of the squares on a four-square board.

When you start the activity, the number of red and black checkers in the beaker is the same. Each time a checker is drawn from the beaker, we will act as if another checker of that color has been added to the beaker. That is, we will assume that before we draw each checker, there are equal numbers of red and black checkers in the beaker.

a) Select one checker **at random** from the beaker. Place the selected checker on square #1 of the four-square board. Since there are two colors of checkers, what is the probability that the color of the checker you drew at random matches the color of square #1 on which it was placed?

Since there are two colors of checkers, the probability that you selected the correct color at random is 1 in 2, or ½.

b) Draw a second checker at random from the beaker and place it on square #2. What is the probability that the color of the second checker you drew matches the color of square #2?

Again, the probability that you selected the correct color at random is 1 in 2, or ½.

c) What is the probability that the colors of both of the checkers you have drawn at random match the colors of the squares on which each was placed?

The total probability is the product of the individual probabilities:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2^2} = \frac{1}{4}$$

d) Draw two more checkers at random, placing the first checker on square #3 and the second checker on square #4.

What is the probability that the colors of all four checkers match the colors of the squares on which they were placed?

The total probability is the product of the individual probabilities:

$$\mathbf{\frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{2^4} = \frac{1}{16}}$$

3) *Combinations of Two Colors on Four Squares*

You can verify your results from part 2) using the 16 four-square boxes shown below.

a) Fill in all possible arrangements of checkers by writing an R for a red checker and B for a black. The first box in each row has been filled in for you.

R	**R**
R	**R**

R	**R**
R	**B**

R	**B**
R	**R**

B	**R**
R	**R**

R	**R**
B	**R**

R	**B**
R	**B**

B	**R**
R	**B**

R	**R**
B	**B**

B	**B**
R	**R**

R	**B**
B	**R**

B	**R**
B	**R**

B	**B**
B	**R**

B	**R**
B	**B**

R	**B**
B	**B**

B	**B**
R	**B**

B	**B**
B	**B**

b) How many different possible arrangements of four checkers are there? _**16**_

c) In how many of these arrangements does the color of each checker match the color of the square on which it is placed? _**1**_

d) Each time you draw four checkers at random and place them on a four-square board, what is the probability that you will draw an ordered arrangement? That is, that the color of each checker will match the color of the square on which it is placed? **_1/16_**

e) How many possible disordered arrangements of four checkers are there? **_15_**

f) What is the probability of drawing a disordered arrangement of checkers? **_15/16_**

g) Does the arrangement of checkers you drew in part 2) match an ordered arrangement or a disordered arrangement? __________________

h) Group Discussion Question: Did anyone draw an ordered arrangement? What is the probability of drawing either an ordered or a disordered arrangement?

4) Probabilities with a Full-Size Checker Board

With the checkers still in place, put the four-square board on the bottom right-hand corner of a full-size checkerboard. Align the four-square board so that the colors of its squares match the square colors on the full-size board.

Now consider the results of filling the entire checkerboard by drawing checkers from the beaker.

a) How many squares does the full-size checkerboard have? **__64__**

b) If you were to fill the entire board by drawing checkers at random, what is the probability that you would get an ordered arrangement of checkers on the full-size checkerboard. That is, what is the probability that the color of each checker would match the color of the square on which it is placed?

$$\frac{1}{2^{64}} = \frac{1}{1.8 \times 10^{19}} = 5.4 \times 10^{-20}$$

c) Generalizing from the checkerboard to other systems, would you expect that it is more common for systems to have orderly or disorderly patterns? Why?

Disorderly patterns are more common because there are many more disorderly patterns than orderly patterns.

d) Does the probability of a disordered system increase as the complexity of the system increases?

Yes. The more complex the system, the greater the probability of a disordered system. In the checkerboard example, the more squares there are to be filled with checkers, the greater the probability that not all of the checkers will match the color of the squares on which they are placed.

5) Entropy

Your instructor will discuss entropy.

a) What is entropy?

Entropy is a measure of the disorder of a system. The greater the disorder, the greater the entropy.

b) Relate the concept of entropy to the 16 possible arrangements of the checkers on the four-square board. Which arrangement corresponds to a low entropy? Which arrangements correspond to higher entropy?

The one arrangement of checkers that matches the colors of the four-square board corresponds to low entropy because this is an ordered arrangement. The other 15 arrangements are disordered, so they correspond to greater entropy.

c) Arrange the pictures of the ice cream man into a sequence of increasing disorder. Which picture corresponds to the greatest disorder?

The picture of the melted puddle of ice cream corresponds to the greatest disorder.

d) Would you have to change the arrangement of the pictures to organize them into chronological order? _**No**_

What happened to the amount of disorder of the ice cream man over time?

As the ice cream man melts, the disorder increases. The system is more disorderly because the ice cream is no longer distinguishable as an ice cream man. The molecules of liquid ice cream are in more disorder than the molecules of solid ice cream because the liquid molecules have greater kinetic energy and move randomly.

What happened to the entropy of the ice cream man over time?

Since the disorder increased, the entropy increased.

e) Is the entropy of the ice cream man likely to decrease on its own? That is, does looking at the pictures in reverse order describe a likely series of events?

No. Even if the melted ice cream were frozen again, it is not likely to freeze into the original ice cream man.

f) If a system can change over time, would you expect the entropy of the system to increase or decrease? Why?

For the ice cream man, like the checkerboard, there are many more disordered arrangements than ordered arrangements. Therefore, an ordered system that changes over time is more likely to change into a disordered system than into another ordered system. Since disorder increases over time, the entropy of systems increases over time.

g) Look at the photos of systems such as the vegetable man or Gary reading a paper. Does the entropy of **all** systems increase over time? Does the entropy increase at the same rate?

In nature, the entropy of a system always increases until the system reaches a state of maximum disorder. However, the rate of entropy increase of some systems is so slow that the increasing disorder is not obvious. In the photos, the vegetable man and the bread sitting in the toaster appear not to change. However, if we were to return one month later, we would notice a change.

6) Entropy and Change of Phase

a) What happens to the motion of water molecules when ice melts into water?

The water molecules leave their fixed positions in the ice crystals and move randomly in the liquid water.

b) What happens to the entropy of ice when it melts into water?

The entropy of the ice increases when it melts because the orderly crystal structure of the ice molecules has become a disordered arrangement of liquid water molecules.

c) For some systems, such as melting ice, it is possible to calculate a numerical value of the change in entropy. How many calories of heat are required to melt 250 grams of ice at 0 ^{o}C into 250 grams of water at 0 ^{o}C? The latent heat of melting of ice is 80 cal/gram.

$$\boldsymbol{Q = L_{heat} \times M = 80\ \text{cal/g} \times 250\ \text{g} = 20{,}000\ \text{cal}}$$

d) What is the change in entropy when the 250 grams of ice at 0 ^{o}C melt into 250 grams of water at 0 ^{o}C?

$$\Delta S = \frac{\Delta Q}{T} = \frac{20{,}000\ \text{cal}}{273\ \text{K}} = 73.3\ \text{cal/K}$$

e) Group Discussion Question: What happens to the entropy of a substance when the substance changes from a liquid to a gas?

6.2 Equilibrium and Entropy

Your instructor will discuss equilibrium. As a system changes with time, it eventually reaches a state of equilibrium. A system in equilibrium does not change over time.

7) Equilibrium Examples - Mixtures

a) Pour light-colored sand on top of the dark-colored sand in the jar so that the light sand forms a layer above the dark sand. Put the cap on the jar and very gently shake the jar. What happens to the layers of light and dark sand?

The two colors of sand become mixed.

b) Continue to shake the jar until the sand is thoroughly mixed. What has happened to the entropy of the sand?

The entropy increased because the ordered layers of sand have become a disordered mixture.

c) If you continued to shake the jar, would the entropy of the sand increase further? Would the entropy decrease?

Once the sand is thoroughly mixed, this system is at its maximum entropy, so the entropy could not increase further. A decrease in entropy would require the sand to separate into layers, which is very unlikely to occur.

d) When a system no longer changes with time, we say that the system is at equilibrium. Is the sand mixture at equilibrium? How can you tell?

Since the sand is unlikely to separate back into layers of light and dark sand, it is at equilibrium.

e) What is the connection between equilibrium and entropy?

Equilibrium is the state of maximum entropy.

8) More Equilibrium Examples

a) Your instructor will show you examples of objects that are not in equilibrium with their surroundings. In the middle column of the table below, explain why the object is not in equilibrium. In the right column, describe what must happen for the object to reach equilibrium with its surroundings.

Object	Cause of Non-Equilibrium	Object at Equilibrium	Entropy Change as Object Moves toward Equilibrium
Inflated balloon	**pressure in balloon is greater than air pressure**	**balloon is deflated**	**A deflated balloon is a more disordered system because the air that was inside the balloon has mixed with air at room pressure.**
Hot cup of coffee	**temperature of coffee is greater than room temp**	**coffee has cooled to room temperature**	**As the coffee cools, thermal energy is transferred to the air and the air heats up. The thermal energy of the coffee is distributed over the coffee and the room, resulting in greater disorder.**
Pile drive mass raised	**mass is not at a stable position because force of gravity pulls down on it**	**mass is at rest on the pile driver stand**	**The gravitational potential energy of the pile driver mass is converted into kinetic, sound, and thermal energy. The energy of the raised pile driver is distributed over the table and the area around the mass.**

Charged capacitor	**positive and negative charges are separated**	**capacitor has discharged (separated charges have come together)**	**The charged capacitor is an ordered system of separated positive and negative charges. A discharged capacitor is a disordered system because the positive and negative charges are mixed.**

b) When does a system reach maximum entropy?

When the system reaches equilibrium with its surroundings, the system is at maximum entropy.

c) A system does not move on its own from an equilibrium state to a non-equilibrium state. For example, a balloon does not inflate itself and a capacitor does not charge itself. Why is this?

The entropy of systems always increases over time. An equilibrium state represents maximum entropy. The entropy of a system left to itself will not decrease.

9) Moving Systems from Equilibrium to Non Equilibrium States – Work In Required

a) It is possible to move a system in equilibrium to a non-equilibrium state by doing work on the system. A system in a non-equilibrium state can store energy.

What action would be required to change each object from its equilibrium state to a non-equilibrium state? What type of energy is stored?

Deflated balloon: **Blowing up a balloon stores strain energy in the walls of the balloon.**

Coffee at room temperature: **Heating up the coffee stores thermal energy.**

Pile driver mass at rest on its stand: **Raising the mass stores gravitational potential energy.**

Discharged capacitor: **Filling the capacitor with separated positive and negative charges stores electrical energy.**

b) Since entropy is always increasing over time, how is it possible to have non-equilibrium systems in nature? How can the entropy of a system on Earth decrease? For example, how can plants grow into ordered structures from the nutrients in the Earth?

While the entropy of the Universe as a whole is always increasing, on Earth we have energy from the Sun, which can be used to do work to move systems from their equilibrium states.

c) Group Discussion Question: What happens to entropy of the dorm room of a typical college student over the course of the academic year? What must be done to the room to decrease its entropy?

10) The Second Law of Thermodynamics

Your instructor will discuss several statements of the second law of thermodynamics. Give examples from the previous activities and demonstrations that illustrate each statement.

a) The entropy of a physical system left to itself will increase or, if the system is already at its maximum entropy, the entropy will remain the same.

The discharged capacitor remains discharged.

b) Any system, when left to itself, tends toward equilibrium with its surroundings.

An inflated balloon deflates, or hot coffee cools to room temperature.

c) The entropy of a system that is in equilibrium with its surroundings remains constant.

The pile driver mass remains at rest on the table.

6.3 Reversible and Irreversible Processes

11) Moving Systems from Non-Equilibrium to Equilibrium – Getting Work Out

We have seen that work must be done on a system to move it from equilibrium to a non-equilibrium state. A system at non-equilibrium stores energy that can be used to do work.

Explain how each of the objects listed below does work as it moves toward equilibrium.

a) Door closer: **Air in the canister is under pressure. When a valve is opened to release the air, the pressure from the air molecules moves a rod attached to the door and the door closes.**

b) Fan Powered by a Heat Engine: **Thermal energy flows from the beaker of hot water into the beaker of cold water until the system comes to thermal equilibrium. This flow of thermal energy generates electricity in a thermocouple, which powers a small fan.**

c) Fan Powered by a Capacitor: **When the capacitor discharges, separated electrical charges in the capacitor come together to form an electric current that powers the fan.**

d) Fan Powered by a Battery: **Chemicals in a battery combine to produce an electric current that powers the fan.**

12) Reversible and Irreversible Processes

Your instructor will discuss reversible and irreversible processes.

a) A discharging capacitor produces an electric current. This current can be used to do work or it could be stored and then used to recharge the capacitor. Could this current be used to recharge the capacitor to its original voltage? Why or why not?

No. Some energy would be wasted as the charges combine to form an electric current. The amount of energy available to recharge the capacitor is less than the electrical energy produced by the capacitor.

b) Is the process of discharging a capacitor reversible?

No. To recharge the capacitor to its original voltage, work must be done on the capacitor.

c) Earlier in this period, you mixed two colors of sand. The system is at equilibrium when the sand is thoroughly mixed. Is the mixing of the sand a reversible or irreversible process?

This is an irreversible process because work must be done on the system of the mixed sand to separate it by color.

d) Another version of the second law of thermodynamics, expressed in terms of irreversible processes, is "All physical processes are irreversible."

How do the following examples illustrate this statement?

Deflated balloon: **A balloon will not inflate by itself. Work must be done on the balloon to blow it up.**

Coffee at room temperature: **Thermal energy must be added to the coffee to heat it up.**

Pile driver mass at rest on its stand: **Work must be done to raise the mass.**

Discharged capacitor: **Electrical energy must be added to the capacitor to recharge it.**

e) During every energy conversion, the entropy of the system increases. Use the concepts of entropy and conservation of energy to explain why reversible processes are not possible.

Conservation of energy tells us that energy cannot be created or destroyed. However, in every energy conversion, the entropy of the system increases, even if only by a small amount. Up to now, we have described this by saying that in every energy conversion, some energy is wasted. Therefore, the amount of energy available to reverse a process is always less than the energy produced by the process.

13) Perpetual Motion Devices?

Your instructor will show you examples of "perpetual motion" devices.

a) Are any of the devices an example of perpetual motion?

No. Some run on batteries and others use magnetic energy. All of the devices will eventually stop.

b) Is the Dippy Duck a perpetual motion device? If not, what is its source of energy?

It is not a perpetual motion device. The dippy duck will stop when the water evaporates from the cup. Thermal energy must continue to heat the duck's head in order for the water to evaporate. Also, the dippy duck draws water from a cup that requires someone to do work to fill the cup.

c) Is it possible to build a perpetual motion machine?

No, the First Law of Thermodynamics (conservation of energy) says that if you take energy out of a system, the system will slow down. Thus, the First Law forbids a perpetual motion machine, in which you get energy out but continue to have the energy available to do additional work.

d) Can a machine run on its own forever without some kind of energy input? Why or why not?

No, perpetual motion does not exist. Since some energy is wasted in every process, that energy must be replaced to keep a machine working. Perpetual motion would violate the second law of thermodynamics.