New Directions for New Dimensions: From Strings to Neutrinos to Axions to ... ?

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1. Introduction: Lowering the fundamental scales of physics

2. String embeddings and the “big picture”
   - hep-ph/9803466
   - hep-ph/9806292
   - hep-ph/9807522

3. Light neutrinos without heavy mass scales: A higher-dimensional seesaw mechanism
   - hep-ph/9811428

4. Extra dimensions and invisible axions
   - hep-ph/9912455

5. Solving the hierarchy problem without SUSY and without extra dimensions...?
   - (in progress)
**Introduction**

Extra spacetime dimensions have been discussed since the original work of Kaluza & Klein in the 1920's...

- original motivation to unify gravity and electromagnetism
- 4D gauge invariance derived from 5D general coordinate invariance!

More recently, extra dimensions have emerged in the context of string theory...

- best candidate for unifying *all* fundamental forces
- at least *six* extra dimensions required

In fact, **two kinds of extra dimensions are possible in string theory**...

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**"Universal" extra dimensions**

These extra dimensions would be experienced by *all* forces

⇒ such extra dimensions would therefore be accessible via accelerator experiments

\[
R^{-1} \geq \mathcal{O}(700 \text{ GeV}) \quad \quad \quad \quad R \leq \mathcal{O}(10^{-19} \text{ m})
\]
Extra dimensions and the MSSM

We imagine that at every point in spacetime, there are extra *compactified* dimensions of radius $R$:

Why don’t we see this experimentally?

If $R^{-1} \geq \mathcal{O}(10^2 \text{ GeV})$, then too small to observe!

Technically, all wavefunctions $\Phi$ must be *periodic* under $y \rightarrow y + 2\pi R$:

$$
\Phi = \sum_{n=0}^{\infty} \phi_n(x) \exp(iny/R)
$$

$\phi_n(x)$ are the "Kaluza-Klein modes", $n \in \mathbb{Z}$.

$$
m_n^2 = m_0^2 + \frac{n^2}{R^2}
$$
Thus, if $m_0 \ll R^{-1}$, we have an infinite tower of Kaluza-Klein modes...

\[ \begin{align*}
    & \quad m_0 + 5/R \\
    & \quad m_0 + 4/R \\
    & \quad m_0 + 3/R \\
    & \quad m_0 + 2/R \\
\end{align*} \]

\[ \begin{align*}
    & \quad m_0 \\
\end{align*} \]

\[ \textbf{excited KK states} \]

\[ \textbf{zero-mode} \]

If accessible energy scale is much less than $R^{-1}$, then we cannot excite higher KK modes!

$\Rightarrow$ Only see KK ground state (zero-mode)!

This is the usual four-dimensional quantum state.

$\Rightarrow$ Threshold for extra dimensions is $\mu_0 \equiv R^{-1}$.

$\Rightarrow$ Extra dimensions are manifested by appearance of Kaluza-Klein modes at higher energy scales $\mu \geq \mathcal{O}(R^{-1})$. 
“Gravity-only” extra dimensions

Recent developments in non-perturbative string theory predict the existence of solitonic “membranes” to which various forces can be restricted — e.g.,

\[ R^{-1} \geq O(10^{-4} \text{eV}) \quad R \leq O(\text{millimeter})! \]
Despite these bounds, extra dimensions have traditionally been considered to be at the Planck scale: \( R \approx 10^{-33} \) cm.

However, this has changed dramatically during the past year. People began to wonder...

- what if the extra dimensions are not so small...?
- what if the corresponding KK states are light...?
- what effects would this have on physics beyond the SM...?

Surprisingly, the answer turned out to be —

Large extra spacetime dimensions have the power to alter the fundamental high energy scales of physics!
These developments have primarily come in three “flavors”...

(1) **Extra dimensions to lower the GUT scale**
- These are $\delta$ universal extra dimensions (“in the brane”).
- They are felt by gauge forces and change the running of the gauge couplings $\alpha_i(\mu)$.
- Unification is preserved, but unification scale is lowered!
- No large hierarchy needed: $M'_{\text{GUT}}/R^{-1} \leq 20$.

(2) **Extra dimensions to lower the Planck scale**
- These are $n$ gravity-only extra dimensions (“off the brane”).
- They change the running of gravitational coupling $G_N(\mu)$.
- This lowers the Planck scale (where $G_N \sim \mathcal{O}(1)$).
- A larger hierarchy is needed: $M'_{\text{Planck}}/r^{-1} \leq 10^{16}$.

(3) **Extra dimensions to lower the string scale**
- These are mixtures of both kinds of extra dimensions (depending on M-theory or Type I realizations).
- The string scale is decoupled from the usual GUT scale and Planck scale, and can be adjusted arbitrarily!
Extra Dimensions and Gauge Coupling Unification

Recall: Without extra dimensions, gauge couplings have usual logarithmic running:

\[ \alpha^{-1}_i(\mu) = \alpha^{-1}_i(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} \]

...result of evaluating the vacuum polarization diagram:
There is even a bit of "experimental" evidence for SUSY grand unification:

**gauge coupling unification!**

![Graph showing three lines representing $\alpha_1^{-1}(\mu)$, $\alpha_2^{-1}(\mu)$, and $\alpha_3^{-1}(\mu)$ as functions of $\log_{10}(\mu/\text{GeV})$.]

Thus, unification scale is $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV.
Now, imagine $\delta = D - 4$ extra dimensions at scale $R^{-1}$. Must also include Kaluza-Klein states in loop!

$\implies$ Below $R^{-1}$, no appreciable effect.
Above $R^{-1}$, couplings now evolve according to:

\[
\alpha_i^{-1}(\mu) \approx \alpha_i^{-1}(R^{-1}) - \frac{b_i - \tilde{b}_i}{2\pi} \ln(R\mu) - \frac{\tilde{b}_i X_\delta}{2\pi \delta} \left[ (R\mu)^{\delta} - 1 \right]
\]


where

- $(b_1, b_2, b_3) \equiv (33/5, 1, -3)$
- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \equiv (3/5, -3, -6)$
- $X_\delta \equiv \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}$

Power-law behavior is the consequence of extra dimensions!
But how does this affect gauge coupling unification?
Let us choose

- $R^{-1} = 1$ TeV (the most extreme case)
- $\delta = 1$ (one extra dimension)

We then find...

* Evolution is dramatically altered, **but gauge couplings still unify!**
* Potential new scale for grand unification!
How does this depend on the chosen radius?

Unification is always preserved!
How does this depend on the number of extra dimensions?

Increasing $\delta$ increases the power-law exponent! Preserves unification, and accelerates it even further!
Thus, we see that extra large dimensions \( \Rightarrow \)

- power-law evolution for gauge couplings
- gauge couplings continue to unify
- unification is weakly coupled and generally perturbative
- unification scale is \textit{flexible} and can be lowered
  (even all the way to TeV, if desired!)

This scenario also has other nice phenomenological features...

- \textbf{cancellation of leading proton-decay diagrams} due to \textit{higher-dimensional} momentum conservation
  (Kaluza-Klein selection rules)
- \textbf{possible explanation of fermion mass hierarchy}
  — small flavor dependence at high scales is \textit{amplified} by power-law running to yield hierarchy at low scales
- \textbf{possible unification even without SUSY}. 
Lowering the Planck scale via extra dimensions

N. Arkani-Hamed, S. Dimopoulos, G. Dvali,
hep-ph/9803315

\[ \tilde{G}_N(\mu) = \mu^2 G_N \]

\( n=2: \)

\( 10^{-4}\text{eV} \quad \text{(TeV)} \quad 10^{19}\text{GeV} \)

\[ r^{-1} \quad \mu^{-2} \quad (2+n)^{-(2+n)} \]

\[ \text{log}(\mu) \]

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How do these three scenarios fit together?

Together, they form an inter-related “circle of ideas”...

But how and why are these scenarios combined?
Embeddings into String Theory

Just as with any GUT theory, we seek an eventual embedding into string theory.

- String theory is automatically consistent with gauge coupling unification.
- String theory provides unification with gravity.
- String theory is finite — resolves issues of non-renormalizability.
- Many other compelling phenomenological features...

But now our fundamental GUT scale is very low!

⇒ Requires a very low string scale!

Is this possible?
• Usually, the string scale is tied to the Planck scale:

\[ M_{\text{string}} \sim g_{\text{string}} M_{\text{Planck}} \]

• But this holds only for **weakly coupled heterotic strings**! As the ten-dimensional coupling increases, this behavior changes due to non-perturbative effects.

\[ \Rightarrow \quad \text{Often, the resulting theory is better described as a Type I (open) string theory!} \]

• For open strings, relation between scales is different:

\[ M_{\text{string}} \sim e^{(\phi)/2} g_{\text{gauge}} M_{\text{Planck}} \]

where

- \( \phi \) is the ten-dimensional **dilaton** field
- \( g_{\text{gauge}} \) is the Type I gauge coupling.

Thus, for **Type I strings**, it is possible to separate the string scale from the Planck scale!
After eliminating the dilaton dependence, one finds that the separation between scales ultimately depends on the volume of compactification:

\[ M_{\text{string}} \sim \sqrt{\frac{1}{\alpha'_{\text{GUT}} M_{\text{Planck}}}} \; V^{-1/4} \]

Now, in our scenario,

- \( R \) and \( \delta \equiv D - 4 \) are input parameters.
- \( M'_{\text{GUT}} \) and \( \alpha'_{\text{GUT}} \) are predicted.
- We choose to set \( M_{\text{string}} = M'_{\text{GUT}} \)

\[ \Rightarrow \text{uniquely fixes the volume } V! \]

Thus, if we write

\[ V \sim R^\delta r^{6-\delta} \]

we can actually solve for the radii of the remaining dimensions!
This is the remaining radius needed to lower \( M_{\text{string}} \) to \( M'_{\text{GUT}} \)!
Most interesting case:

One extra dimension at $R \approx 0.5$ TeV

$\implies M_G^U \approx 10$ TeV

$\implies$ remaining five dimensions must have radius

$r \approx (10 \text{ MeV})^{-1} \approx 10$ fermi

So large! Seems impossible!

But these extra dimensions are felt only by gravity!
(Gauge forces are immune to their effects.)

$\implies$ Not excluded by any laboratory, astrophysical, or cosmological constraints!

Thus, we are led to a unified embedding into string theory:

\[ \alpha_1^{-1}, \quad \alpha_2^{-1}, \quad \alpha_3^{-1} \]

\[ \text{Type I'} \text{ string theory} \]

\[ \alpha_{\text{string}}^{-1} \]

\[ \text{gauge: 4D, 4D, 5D, 10D} \]

\[ \text{gravity: 4D, 9D, 10D, 10D} \]

\[ \log \mu \]

\[ \text{10 MeV, } \frac{1}{r'}, \quad 100 \text{ GeV, } \frac{1}{M_Z}, \quad 0.5 \text{ TeV, } \frac{1}{R}, \quad 10 \text{ TeV, } M'_{\text{GUT}} \]

\[ \Rightarrow \quad \text{All of the fundamental scales (GUT, string, Planck) have been lowered simultaneously!} \]
This makes sense, since string theory is a theory of both gauge forces and gravitational forces...

• This reduced-scale “structure” survives regardless of the amount by which the fundamental energy scales are lowered.

Thus, we see that extra spacetime dimensions have succeeded in lowering all of the fundamental high-energy scales of physics in a self-consistent way...
This then leads to a new view of spacetime:

- At low energies, the (MS)SM lives on the four-dimensional intersection of different "branes".
- At higher energies, the gauge forces feel the extra dimension of radius $R \rightarrow$ world becomes a cylinder! This lowers GUT scale, ensures gauge coupling unification.
- Remaining orthogonal directions felt only by gravitational interactions! This lowers the Planck scale, explains weakness of gravity.
- Lowered string scale: ensures consistency, calculability.
This also opens up whole new vistas that await exploration!

**Particle Theory:**

Many aspects of physics beyond the SM must now be considered in a new light:

- SUSY
- SUSY-breaking
- GUT physics
- String theory

{ role of extra large dimensions? effects of changed energy scales?}

**Collider Experiments:**

If scales are extrapolated to TeV-range, many striking signals are possible at future colliders!

- direct detection of Kaluza-Klein states
- decays of KK states into (s)fermions
- Drell-Yan production of KK states in $p\bar{p}$ collisions
- new effective four-fermi contact interactions via exchange of gauge-boson KK states
- direct detection of GUT particles, Regge states, quantum gravity...?

⇒ Can probe properties of GUTs and strings *experimentally*!
⇒ A new *experimental* direction for string phenomenology?
Cosmology:

Profound effects for cosmology!
Fundamental energy scales have changed!

- role of light stable KK states
- possible new dark-matter candidates
- reheating and thermal regeneration of KK states
- inflation in higher dimensions
- phase transitions in higher dimensions
- topological defects
- density perturbations
- cosmological methods for generating large radii?
- possible explanation for dimensionality of spacetime??
- ... even other wilder ideas ...

Lots to think about!
However, the important point is that

- extra dimensions can be taken seriously as physical entities
- they can have measurable, significant effects

The "fundamental" high energy scales of physics are not immutable.

The "parameter space" of ideas for physics beyond the Standard Model may be significantly broader than we previously expected.

These are undoubtedly the most valuable lessons.
So, moral of the story seems to be

HIGH FOUR-DIMENSIONAL SCALES

are replaced by

LOWER HIGHER-DIMENSIONAL SCALES!

But there are still other scales
eg. seesaw scale!
Generating Light Neutrino Masses

Recall usual SO(10) seesaw mechanism...

- $SO(10) \implies \text{right-handed neutrino } N$
- Assume Yukawa coupling $y_{\nu}^{\nu} H_{+} N$
  $\implies \text{Dirac mass } m \equiv y_{\nu} \langle H_{+} \rangle \approx \mathcal{O}(10^{2}) \text{ GeV}$.
- Also assume Majorana mass for $N$
  (can arise from GUT breaking via 126 vevs)
  $\implies M \approx \mathcal{O}(10^{16}) \text{ GeV}$.
- Total mass terms then take form

$$\begin{pmatrix} \nu_{L} & N \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu_{L} \\ N \end{pmatrix} \quad \text{where } \mathcal{M} \equiv \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}.$$  

- Diagonalizing to find mass eigenvalues, we obtain
  $$\lambda_{-} \approx -\frac{m^{2}}{M}, \quad \lambda_{+} \approx M.$$  

- Lightest eigenvalue "seesaws" against heavy mass scale!
  $$m_{\nu} \approx 10^{-2} \text{ eV} \implies M \approx 10^{14-16} \text{ GeV}.$$  

Thus, light neutrino masses seem to provide further evidence for a high fundamental GUT scale!
Light Neutrinos without Heavy Mass Scales


Fundamental observation:

Unlike all other fields, right-handed neutrino is SM singlet!

- not restricted to brane like other SM particles
- can propagate in higher-dimensional "bulk"
- can have infinite tower of KK excitations!

This can have many effects on the resulting neutrino mass...
(1) **Power-law running of neutrino Yukawa couplings...**

\[ \begin{align*}
& \Rightarrow \text{neutrino Yukawa coupling driven to very small values over short energy interval} \\
& \Rightarrow \text{neutrino mass is power-law suppressed relative to masses of all other fermions!}
\end{align*} \]

(2) **Suppression by bulk volume factor...**

- if \( N \) field is in bulk, its wavefunction must be renormalized by bulk volume factor
- suppresses Dirac coupling \( m \) by factors \( (RM_s)^n \).

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*Both of these mechanisms suppress \( m_\nu \) by suppressing \( m \).*

**But is there a higher-dimensional analogue of the seesaw mechanism?**
• Consider $D = 5$ for concreteness:
  $\Rightarrow$ bulk field $\Psi \equiv (\psi_1, \bar{\psi}_2)^T$ in Weyl rep.

• Assume lepton-number conserved on brane:
  $\Rightarrow$ no primordial $\nu_L \nu_L$ mass term
  (just as in usual seesaw mechanism)

• Allow lepton-number breaking in bulk:
  $\Rightarrow$ “bare” Majorana mass term $\frac{1}{2} M_0 \bar{\Psi} \Psi$.

• Allow general Dirac couplings $(m_1, m_2)$
of brane field $\nu_L$ to bulk fields $(\psi_1, \psi_2)$.

• Expand $\psi_{1,2}$ in KK modes:
  
  $\psi_1(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_1^{(n)}(x) \cos(ny/R)$
  
  $\psi_2(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \psi_2^{(n)}(x) \sin(ny/R)$.

  $\Rightarrow$ no zero-mode for $\psi_2$.

• For $n \geq 1$, define $\bar{N}^{(n)} \equiv (\psi_1^{(n)} + \psi_2^{(n)})/\sqrt{2}$
  $M^{(n)} \equiv (\psi_1^{(n)} - \psi_2^{(n)})/\sqrt{2}$.
Then $\nu_L$ mixes with entire spectrum of KK states!

Basis: \( (\nu_L, \psi_1^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, \ldots) \)

Mass mixing matrix takes the form:

\[
\begin{pmatrix}
0 & m & m_N & m_M & m_N & m_M & \cdots \\
\text{m} & M_0 & 0 & 0 & 0 & 0 & 0 \\
m_N & 0 & M_0 + \frac{1}{R} & 0 & 0 & 0 & \cdots \\
m_M & 0 & 0 & M_0 - \frac{1}{R} & 0 & 0 & \cdots \\
m_N & 0 & 0 & 0 & M_0 + \frac{2}{R} & 0 & \cdots \\
m_M & 0 & 0 & 0 & 0 & M_0 - \frac{2}{R} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

This leads to many unexpected higher-dimensional features depending on parameters $m$, $m_N$, $m_M$, $M_0$, and $R$...
Simplest case ("orbifold case"): 

- Assume SM brane located at orbifold fixed point \( y = 0 \) 
  \[ \Rightarrow \] then no coupling \( \nu_L \psi_2 \), since \( \psi_2 = 0 \) on brane 
  \[ \Rightarrow \] \( m_{M} = m_{N} = m \).

- Assume \( M_0 = 0 \) (lepton-number conserved in bulk).

Then mass matrix takes the form

\[
\begin{pmatrix}
0 & m & m & m & m & \ldots \\
m & 0 & 0 & 0 & 0 & \ldots \\
m & 0 & 1/R & 0 & 0 & \ldots \\
m & 0 & 0 & -1/R & 0 & \ldots \\
m & 0 & 0 & 0 & 2/R & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
\end{pmatrix}
\]
Eigenvalues are solutions to $\lambda R = \pi (m R)^2 \cot(\pi \lambda R)$.

All eigenvalues paired (Dirac masses only)
- for $mR \ll 1$, find $m_\nu \approx m$ (no seesaw behavior)
- for $mR \gg 1$, find $m_\nu \approx (2R)^{-1}$
  - independent of Yukawa coupling $m$!
  - neutrino mass bounded by the radius!
But is the actual value of $m_{\nu}$ important for oscillations?

Consider oscillations between $\nu_L$ and KK states

(analogue of usual neutrino/anti-neutrino oscillations).

If $U$ diagonalizes the mass matrix, then

$$P_{\nu_L \rightarrow \nu_L}(t) = \left| \sum_k |U_{\nu_k}|^2 \exp\left( \frac{i \lambda_k^2 t}{2p} \right) \right|^2.$$

We then find...

- Neutrino oscillations still effectively periodic.
- Deficits and regenerations are never total!
- Oscillation length set by first eigenvalue interval $\approx R^{-1}$ rather than by $m_{\nu}$ itself!

$$\delta m^2 \approx 10^{-4} \text{ eV}^2 \implies R \approx 10^{-5} \text{ meters}!$$
Bare Majorana case:

Consider effects of bare Majorana mass $M_0$ in bulk.

Note: we expect $M_0 \approx M_s \gg R^{-1}$, hence $m_\nu \sim O(m^2/M_0)$.

\[
\begin{pmatrix}
0 & m & m & m & m & \ldots \\
m & M_0 & 0 & 0 & 0 & \ldots \\
m & 0 & M_0 + \frac{1}{R} & 0 & 0 & \ldots \\
m & 0 & 0 & M_0 - \frac{1}{R} & 0 & \ldots \\
m & 0 & 0 & 0 & M_0 + \frac{2}{R} & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

But now define $\epsilon \equiv M_0 \pmod{R^{-1}}$ (smallest diagonal entry)

$\Rightarrow$ all other diagonal entries are $\epsilon \pm k'/R$...

\[
\begin{pmatrix}
0 & m & m & m & m & \ldots \\
m & \epsilon & 0 & 0 & 0 & \ldots \\
m & 0 & \epsilon + \frac{1}{R} & 0 & 0 & \ldots \\
m & 0 & 0 & \epsilon - \frac{1}{R} & 0 & \ldots \\
m & 0 & 0 & 0 & \epsilon + \frac{2}{R} & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

Heavy scale $M_0$ decouples from the physics!

$\Rightarrow$ only scale remaining is $R^{-1}$

$\Rightarrow$ resulting (Majorana) neutrino mass is $m_\nu \sim O(m^2 R)$.

The KK seesaws have replaced $M_0$ with $R^{-1}$!
Scherk-Schwarz special case:

- One possible way to generate \( M_0 \neq 0 \) is to break lepton-number in the bulk via the Scherk-Schwarz mechanism:

\[
\psi_{1,2}(2\pi R) = -\psi_{1,2}(0).
\]

- This is a global (non-local) breaking of lepton-number.

- In such cases, we then find

\[
M_0 = \frac{1}{2} R^{-1}
\]

This precise value is fixed topologically.

\[
\begin{pmatrix}
0 & m & m & m & \cdots \\
1/(2R) & 0 & 0 & 0 & \cdots \\
0 & -1/(2R) & 0 & 0 & \cdots \\
0 & 3/(2R) & 0 & \cdots & \cdots \\
0 & 0 & -3/(2R) & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \cdots
\end{pmatrix}
\]
Eigenvalues are solutions to \[ \lambda R = -\pi (mR)^2 \tan(\pi \lambda R) \]

- Tangent curves have been shifted by half-period
  \[ \Rightarrow \text{neutrino is exactly massless!} \]
  \[ \Rightarrow \text{result holds for all values for } mR! \]
  \[ \Rightarrow \text{all other KK states have } \text{Dirac masses!} \]

- Neutrino mass eigenstate is exactly given by
  \[ |\tilde{\nu}_L\rangle \sim |\nu_L\rangle - mR \sum_{k=1}^{\infty} \frac{1}{k-1/2} \left\{ |N^{(k-1)}\rangle - |M^{(k)}\rangle \right\} \]
  \[ \Rightarrow \text{mostly } |\nu_L\rangle \text{ for } mR \ll 1, \text{ as required!} \]
  \[ \Rightarrow \text{this combination cancels neutrino mass exactly!} \]
But what about neutrino oscillations?

\[ P_{\nu_L \rightarrow \nu_L}(t) = \left| \sum_k |U_{\nu k}|^2 \exp \left( \frac{i \lambda_k^2 t}{2p} \right) \right|^2. \]

Note: \( M \)-matrix non-diagonal \( \Rightarrow \) \( U \)-matrix non-diagonal.

Thus, still have oscillations!

- Neutrino oscillations without neutrino masses in \( D \geq 5 \).
- Oscillations still effectively periodic.
- Regenerations never total, but deficits total.
- Oscillation length set by first eigenvalue interval \( \approx R^{-1} \).
- Easy to generalize to flavor oscillations, even with \((\nu_e, \nu_\mu, \nu_\tau)\) massless! Oscillations indirect via KK states...

Thus, in such scenarios, neutrino oscillations are evidence not for neutrino masses, but for extra spacetime dimensions!
Finally, in Type I string theory, there also exists a new way to break lepton number...

- Thus far, assumed SM brane at $y = 0$ (fixed point).
- But in Type I string theory, can shift branes away from fixed points to arbitrary location $y^* \neq 0$.

Brane/bulk coupling then takes the form

$$m \bar{\nu}_L \left( \Psi + \Psi^c \right) \bigg|_{y=y^*} + \text{h.c.} \quad \text{where} \quad \Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix}$$

$\Rightarrow$ generates a coupling between $\nu_L$ and $\psi_2$!
$\Rightarrow$ establishes unequal Dirac couplings $m_N \neq m_M$ in seesaw matrix:

$$m_N^{(n)} = m \left[ \cos(ny^*/R) + \sin(ny^*/R) \right]$$

$$m_M^{(n)} = m \left[ \cos(ny^*/R) - \sin(ny^*/R) \right].$$

- This splits the Dirac masses into unequal Majorana masses:

$$\lambda_{\pm} = \frac{1}{2} \left[ \mu \pm \sqrt{\mu^2 + 4m^2} \right], \quad \text{where}$$

$$\mu = -m^2 R \sum_{k=1}^{n} \frac{1}{k} \sin \left( \frac{2ky^*}{R} \right).$$

Thus, brane-shifting is a uniquely stringy way of breaking lepton-number, establishing a seesaw, and generating a Majorana neutrino mass.
Extension to flavor ...

⇒ three LH νi on brane

**Bulk?**

- three corresponding bulk neutrinos
- 3×3 mixing matrices for brane/bulk couplings

- Mohapatra & Perez-Lorenzana
- Barbieri, Creminelli, Strumia

Lots of parameters!

ANOTHER ALTERNATIVE?
A "compact" model

- three left-handed $\nu$ on brane (flavors)
- "bare" Majorane masses $m_\nu$
  BUT no mixing!
- only one bulk neutrino
- universal brane/bulk coupling

$$\begin{pmatrix}
  m_1 & 0 & 0 \\
  m_2 & m & m \\
  m_3 & m & m \\
  & & & & \ddots
\end{pmatrix}$$

$$\begin{pmatrix}
  m & m & m & m & m \\
  m & m & m & m & m \\
  m & m & m & m & m \\
  & & & & \ddots
\end{pmatrix}$$

$$\tan \Delta \text{AR} = \frac{1}{\frac{1}{2} - m_1}$$

eigenvalues:

⇒ still yields flavor oscillations $\nu_i \leftrightarrow \nu_j$
  indirectly through KK states!

Thus, flavor oscillations $\nu_i \leftrightarrow \nu_j$ are possible \textbf{even if}

⇒ brane theory (SM) is flavor-diagonal
⇒ bulk theory is flavor-neutral
⇒ brane/bulk coupling is flavor-blind!

• ONLY five parameters ⇒ all masses & "mixings"!
Thus, with extra large dimensions, it is possible to...
- generate light neutrinos via power-law Yukawa running
- suppress neutrino masses via higher-dim. volume factors
- establish seesaw mechanism via towers of KK states
- break lepton-number via Type I brane shifting
- have neutrino oscillation wavelengths set by radii
- have neutrino oscillations without neutrino masses!

Moreover, these scenarios can easily be generalized to
- multiple extra spacetime dimensions with different radii
- different $\Psi$ fields in different directions
- indirect flavor oscillations via flavor/KK non-diagonality.

Rich neutrino phenomenology is possible in extra dimensions!

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Of course, these scenarios are at best only qualitative.

- Still must perform detailed experimental comparisons
  - atmospheric neutrinos vs. solar neutrinos, ...
  - effects of KK admixtures ($e.g., \tau \rightarrow \mu \bar{\nu}_\mu$).

- Theoretical issues:
  - incorporate mechanisms within realistic string models
  - brane dynamics? (branes not truly rigid)
  - other methods of protecting/breaking lepton-number.

Many ideas, but further study needed!