A Lot of Flavor Physics from a Little Symmetry

Richard Lebed
Arizona State University/
Jefferson Lab

DPF 2000
August, 2000
"U(2) flavor physics without U(2) symmetry"

"Maximal neutrino mixing from a minimal flavor symmetry"

Alfredo Aranda, Christopher Carone (William & Mary),
RFL
Outline

1) Flavor symmetry: Why and which?

2) Discrete symmetries

3) Successful textures: The $U(2)$ model

4) Our model: $T' \times \mathbb{Z}_3$
The case for seeking a flavor symmetry

Family replication problem:

Begins in 1947 with discovery of Ti meson and that
the μ is not strongly interacting, hence redundant

Now, half a century later, we have 3 complete generations

\[
\begin{pmatrix}
u_e \\ d_e \\
\end{pmatrix} \begin{pmatrix}
u_e \\ s_e \\
\end{pmatrix} \begin{pmatrix}t_e \\ b_e \\
\end{pmatrix} \begin{pmatrix}
u_e \\ e \\
\end{pmatrix} \begin{pmatrix}
u_\mu \\ \mu \\
\end{pmatrix} \begin{pmatrix}
u_\tau \\ \tau \\
\end{pmatrix}
\]

color \(\alpha = 1,2,3\)

--- Assertion: Redundancy implies symmetry ---

In this case, variously called a flavor, family, generational, or
horizontal symmetry

\(G_f\)

However, \(G_f\) must be a badly broken symmetry:

\[
\frac{m_t}{m_\mu} \sim 40,000, \quad |V_{ub}| \sim 3 \times 10^{-3}
\]

⇒ It must have a hierarchical breaking pattern to
accommodate several distinct magnitudes
Requirements for a successful Gf

1) Accommodate strongly hierarchical quark and charged lepton masses

2) Accommodate strongly hierarchical Cabibbo-Kobayashi-Maskawa (CKM) matrix

3) Suppress flavor-changing neutral current (FCNC) effects so that K¯¯ mixing, μ→eγ, etc. do not occur at phenomenologically unacceptable levels

   In particular, FCNCs are well studied in supersymmetry, which is also stable under radiative corrections. In completely minimal SUSY, K¯¯ mixing is suppressed by near degeneracy of d and s squark masses

4) Accommodate neutrino mass and mixing data from Super-K, CHOOZ, SAGE, GALLEX, ...

5) Suppress proton decay

   However, 1) - 4) can be studied using only Gf field content and basic symmetry-breaking information. 5) done properly requires detailed assumptions about ultrahigh-energy dynamics, so we do not consider it further
What kind of horizontal symmetry?

- Fundamental global symmetries are said to be violated by quantum gravity effects, and are believed to be incompatible with string theory.

  S. Giddings, A. Strominger  
  NPB 307 (1988) 854

  S. Coleman  
  NPB 310 (1988) 613

  G. Gilbert  
  NPB 328 (1989) 159

So we consider only gauged symmetries.

- In SUSY, continuous gauged symmetries tend to give rise to D-term interactions between flavons and sparticles that introduce excessive FCNCs.

  Y. Kawamura, H. Murayama, M. Yamaguchi  
  PRD 51 (1995) 1357

So we consider only discrete gauged symmetries. They are also smaller - groups with only a finite number of elements.

* K.S. Babu and Mohapatra  
  PRL 83 (1999) 2522  suggest a way to evade such interactions.
Does "discrete gauge symmetry" make sense?

- Without continuously connected group elements, covariant derivatives cannot be defined in the usual way

⇒ No gauge bosons

Then is a gauged discrete symmetry the same as a global discrete symmetry?

NO: The gauge case still has nontrivial topological information

In particular, cosmic string solutions exist, evidence for which would arise in scattering processes via Aharanov-Bohm phases

[Krauss and Wilczek, PRL 62, 1221 (1989)]

Moreover, gauge invariance restricts possible terms appearing in the Lagrangian, and configurations equivalent up to a gauge transformation can be identified in unitary gauge

Essential features of discrete groups

- Finite groups have a finite number of inequivalent irreducible matrix representations (reps)
  - (one-dimensional: singlet, two-dimensional: doublet, etc.)

- Reps of dimension $>1$ appear ONLY in non-Abelian groups

  - Essential to have doublets to protect degeneracy of sparticles in first two generations

On the other hand, 3rd generation Yukawa couplings, particularly $h_t$, appear to be distinguished

$\Rightarrow$ Suggests $2 + 1$ structure for matter fields
Anomaly cancellation conditions

The only conditions relevant at low energy are those linear in the high-scale symmetry $G_5$ and quadratic in low-scale, non-Abelian symmetries

[Ibáñez and Ross, PLB 260, 291 (1991);
Banks and Dine, PRD 45, 1424 (1992)]

- All other anomaly constraints may be satisfied by introducing additional heavy fields with minimal low-energy consequences.

In practice, we implement the linear constraint by demanding fermion fields that couple to $SU(3)_c$ or $SU(2)_w$ to appear in $G_f$ multiplets that fill complete multiplets of an anomaly-free continuous embedding group.

Finally, one $U(1)$ factor or $Z_2$ subgroup of $G_f$ may be left anomalous, its effects accommodated by the Green-Schwarz mechanism.
The observed charged fermion textures

Let $\lambda \sim \sin \theta_c = 0.22$

Then (assuming for definiteness a SUSY GUT and working at the GUT scale)

$$ht: h_c: h_u \sim 1: \lambda^4: \lambda^8,$$

$$h_b: h_s: h_d \sim 1: \lambda^2: \lambda^4,$$

$$h_t: h_{\mu}: h_e \sim 1: \lambda^2: \lambda^4,$$

and

$$\frac{m_b}{m_t} \sim 1, \quad \frac{m_b}{m_t} \sim \lambda^3,$$

$$V_{us} \sim \lambda, \quad V_{ub} \sim \lambda^3 - 4, \quad V_{cb} \sim \lambda^2$$

FCNCs:

$$\frac{\tilde{m}_1 - \tilde{m}_2}{\frac{1}{2} (\tilde{m}_1 + \tilde{m}_2)} \lesssim \lambda^4$$

$$\Rightarrow$$ Assign first two generations to a doublet rep (non-Abelian group)
and the third to a singlet (a $2 + \frac{1}{2}$ model)

Such models explain $m_u << m_c << m_t$ but not $m_b << m_t$

[Alternatives: 1) Triplet reps with very large, $(\Theta(1))$ symmetry breaking

D.B. Kaplan, M. Schmaltz, PRD49 (1994) 3741

2) Only $t$ mass a singlet of $G_f$; $b, \tau$ after symmetry broken

P.H. Frampton, O.C.W. Kong PRL 77 (1996) 1699

L.J. Hall, H. Murayama PRL 75 (1995) 3985, ...
The U(2) model

Barbieri, Dvali, Hall, PLB 377, 76 (1996)

$G_f = \text{U}(2)$ provides a natural symmetry for $2 \otimes 1$ family structure:

Yukawa matrices

\[
\begin{pmatrix}
S_{ab} + A_{ab} & \phi^a \\
\phi_a & 1
\end{pmatrix}
\]

where $S, A$ are $2 \times 2$ symmetric, antisymmetric tensors ($3$ and $1$ of $U(2)$)
$\phi^a$ is a two-component column vector ($2$ of $U(2)$)

Symmetry breaking:

$U(2) \xrightarrow{\varepsilon} U(1) \xrightarrow{\varepsilon'} \text{Nothing} , \quad \varepsilon' \ll \varepsilon \ll 1$

\[
\frac{\langle \Phi \rangle}{M_f} \sim \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix} , \quad \frac{\langle A \rangle}{M_f} \sim \begin{pmatrix} 0 & \varepsilon' \\ -\varepsilon' & 0 \end{pmatrix} , \quad \frac{\langle S \rangle}{M_f} \sim \begin{pmatrix} 0 & 0 \\ 0 & \varepsilon \end{pmatrix}
\]

Gives

\[
Y_{U,D,L} \sim \begin{pmatrix}
0 & \varepsilon' & 0 \\
-\varepsilon' & \varepsilon & \varepsilon \\
0 & \varepsilon & 1
\end{pmatrix} \Rightarrow \text{Eigenvalues} \quad \frac{\varepsilon'^2}{\varepsilon} , \varepsilon , 1
\]

An essential feature: $2 \otimes 2 = 3 \otimes 1$

(Else, upper $2 \times 2$ block would develop large entries besides $Y_{22}$)
The group $T'$

- Double tetrahedral group (24 elements): Symmetry group of proper rotations of a regular tetrahedron in $SU(2)$ double-covering of $SO(3)$

- The smallest group with singlets, doublets, triplets, and the rule

\[ 2 \otimes 2 = 3 \oplus 1 \]

- Reps: \(1^0, 1^+ , 1^- , 2^0 , 2^+ , 2^- , 3\)

"Triality" superscripts add modulo 3

e.g., \(2^+ \otimes 2^- = 3 \oplus 1^0\)

<table>
<thead>
<tr>
<th>$SU(2)$</th>
<th>$T'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1^0$</td>
</tr>
<tr>
<td>2</td>
<td>$2^0$</td>
</tr>
<tr>
<td>3</td>
<td>$3$</td>
</tr>
<tr>
<td>4</td>
<td>$2^+ \oplus 2^-$</td>
</tr>
<tr>
<td>5</td>
<td>$3 \oplus 1^+ \oplus 1^-$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Protecting the 1st generation

In order to allow an entry in $Y_{22}$ but not $Y_{11}, Y_{12}, Y_{21}$ (so $m_c \gg m_u$, etc.), $G_f$ must have elements that change 1st generation entries but leave $Y_{22}$ alone, i.e.,

$$g = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

$T'$ has such elements, but in a rep $(2^-)$ not anomaly free.

On the other hand, anomaly-free $Z_2$ has an element = diag($e^{-2\pi i/3}$, $e^{2\pi i/3}$)

⇒ Minimal choice: Introduce extra $Z_3$ to soak up phase

$$G_f = T' \times Z_3$$

is the smallest group that reproduces $U(2)$ textures

$$G_f = T' \times Z_3 \xrightarrow{\epsilon} H_f = (Z_3 \times Z_3)^{diag} \xrightarrow{\epsilon'} \text{Nothing}$$

Label $Z_3$ reps by superscript: In $G_f = T' \times Z_3$, one has reps

$1^0, 1^{0+}, 1^{-}, 1^+, 1^{++}, 1^{+-}, 1^-, 1^{--},$

$2^0, 2^{0+}, 2^{-}, 2^+, 2^{++}, 2^{+-}, 2^-, 2^{--},$

$3^0, 3^-.$
Model Charge Assignments

\[ Q, U, D, L, E \sim 2^0 \oplus 1^0 \]
\[ H_u, D \sim 1^0 \]
\[ Y_{u, d, l} \sim \left( \begin{array}{c|c}
3^- & 2^0 \\
\hline
2^0 & 1^0 \\
\end{array} \right) \]

Take flavors

\[ \phi \sim 2^0, \quad A \sim 1^0, \quad S \sim 3^- \]

\[ T' \times \mathbb{Z}_3 \xrightarrow{\langle \phi^2 \rangle \sim 3} \mathbb{Z}_3^{\text{Diag}} \xrightarrow{\langle A \rangle \sim 3'} \text{Nothing} \]

- Gives precisely same texture as U(2) model
- Can now include GUT quantum numbers as before to obtain detailed phenomenological fits
Neutrinos

- Use seesaw mechanism to achieve small neutrino masses:

\[
\begin{align*}
M_{LR} & : \text{Neutrino Dirac mass matrix} \\
M_{RR} & : \text{Neutrino Majorana mass matrix (heavy)} \\
M_{LL} & \approx M_{LR} M_{RR}^{-1} M_{LR}^T
\end{align*}
\]

Then

\[
\begin{array}{cccc}
\nu_L & M_{LR} & (B-M_{RR})^{-1} & M_{LR}^T \\
\times & \times & \times & \nu_{L'}
\end{array}
\]

Data from KEK, SAGE, GALLEX, CHOOZ (but not LSND) suggest 3-neutrino mixing with

\[
\begin{cases}
\sin^2 2\theta_{23} > 0.8 \\
10^{-3} \text{ eV}^2 \leq \Delta m_{23}^2 \leq 10^{-2} \text{ eV}^2
\end{cases}
\]

atmospheric neutrino deficit

and EITHER

\[
\begin{cases}
2 \times 10^{-3} \leq \sin^2 2\theta_{12} \leq 10^{-2} \\
4 \times 10^{-6} \text{ eV}^2 \leq \Delta m_{12}^2 \leq 10^{-5} \text{ eV}^2
\end{cases}
\]

Solar neutrino deficit

Small-angle MSW solution

OR

\[
\begin{cases}
\sin^2 2\theta_{12} > 0.7 \\
6 \times 10^{-6} \text{ eV}^2 \leq \Delta m_{12}^2 \leq 5 \times 10^{-5} \text{ eV}^2
\end{cases}
\]

large-angle MSW solution

Not considered here: Vacuum oscillation

sterile neutrinos
Let $v_R \sim 2^- \oplus 1^+$

And introduce $\phi_v \sim 2^{+0} \rightarrow (\bar{\epsilon}, \bar{\epsilon})$ Note that $U(2)$ has only one $\bar{\epsilon}$

For the GUT case, an extra $\phi_v$ is needed to avoid a singular $M_{RR}$

Then $M_{RR} \sim \langle H_D \rangle \begin{pmatrix} 3^- & 1^{0-} & 2^{+0} \\ \bar{1}^{0+} & \bar{1}^{1-} & \bar{1}^{1+} \end{pmatrix} \sim \langle H_D \rangle \begin{pmatrix} 0 & \bar{\epsilon}' & \bar{\epsilon}' \\ -3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ 0 & \bar{\epsilon} & 0 \end{pmatrix}$

$M_{RR} \sim \Lambda_R \begin{pmatrix} 3^- & 2^{+0} \\ \bar{2}^{+0} & \bar{1}^{1-} \end{pmatrix} \sim \Lambda_R \begin{pmatrix} \epsilon^2 & \epsilon \bar{\epsilon}' & \epsilon' \\ \epsilon \bar{\epsilon}' & \epsilon^2 & \epsilon \\ \epsilon' & \epsilon & 0 \end{pmatrix}$

$\Rightarrow M_{ll} \sim \langle H_D \rangle^2 \frac{\Lambda_R}{\Lambda_R} \begin{pmatrix} (\epsilon' / \epsilon)^2 & \epsilon' / \epsilon & \epsilon' / \epsilon \\ \epsilon' / \epsilon & 1 & 1 \\ \epsilon' / \epsilon & 1 & 1 \end{pmatrix}$

Results: Mass eigenvalues in ratio $1 : \epsilon' : \epsilon'$

23 mixing angle is $\Theta(1)$

12 mixing angle is $\Theta(\epsilon'_\epsilon)$
\[ Y_u = \begin{pmatrix} 0 & u_1^* \rho & 0 \\ -u_1^* \rho & u_2 \rho & u_3 \xi \\ 0 & u_4 \xi & u_5 \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & d_1 \xi' & 0 \\ -d_1 \xi' & d_2 \xi & d_3 \xi \\ 0 & d_4 \xi & d_5 \end{pmatrix} \]

\[ Y_L = \begin{pmatrix} 0 & c_1 \xi' & 0 \\ -c_1 \xi' & 3 c_2 \xi & c_3 \xi \\ 0 & c_4 \xi & c_5 \end{pmatrix} \]

\[ M_{\nu_R} = \begin{pmatrix} 0 & l_1 \xi' & l_3 \xi \xi' \\ -l_1 \xi' & l_2 \xi & l_3 \xi \\ 0 & l_4 \xi & 0 \end{pmatrix} \begin{pmatrix} \langle H_0 \rangle \\ \nu_R \end{pmatrix}, \quad M_{\nu_R} = \Lambda_R \begin{pmatrix} r_4 r_2 \xi^2 & r_4 r_1 \xi \xi' & r_2 \xi' \\ r_4 r_1 \xi \xi' & r_3 \xi & r_1 \xi \\ r_2 \xi' & r_1 \xi & 0 \end{pmatrix} \]

- One loop RGE run from $M_{\nu_R}$ down to $m_\tau$
- Solve for mass eigenvalues, mixing angles
- $\chi^2$ fit, with coefficients constrained to be close to unity
### TABLES

\[ \epsilon = 0.04, \ \rho = 0.08, \ \epsilon' = 0.004, \ \xi = 0.017 \]

| \(c_1\) = \(-0.93 \pm 0.01\) | \(d_1\) = \(+1.33 \pm 0.45\) | \(l_1\) = \(+0.85 \pm 0.62\) | \(r_1\) = \(+0.94 \pm 0.84\) | \(u_1\) = \(+0.92 \pm 0.31\) |
| \(c_2\) = \(-0.46 \pm 0.03\) | \(d_2\) = \(-0.81 \pm 0.26\) | \(l_2\) = \(-1.01 \pm 1.11\) | \(r_2\) = \(+1.06 \pm 0.95\) | \(u_2\) = \(+1.48 \pm 0.70\) |
| \(c_3\) = \(-1.02 \pm 1.13\) | \(d_3\) = \(+1.55 \pm 0.67\) | \(l_3\) = \(-0.97 \pm 0.75\) | \(r_3\) = \(+1.03 \pm 1.12\) | \(u_3\) = \(-0.90 \pm 0.91\) |
| \(c_4\) = \(-1.03 \pm 1.15\) | \(d_4\) = \(+1.14 \pm 1.33\) | \(l_4\) = \(-1.09 \pm 1.04\) | \(r_4\) = \(-1.07 \pm 1.05\) | \(u_4\) = \(+1.07 \pm 1.21\) |
| \(c_5\) = \(-0.90 \pm 0.01\) | \(d_5\) = \(-1.29 \pm 0.12\) | \(l_5\) = \(-1.11 \pm 0.79\) | \(r_5\) = \(-0.97 \pm 1.03\) | \(u_5\) = \(+1.84 \pm 0.95\) |

| \(a\) = \(+0.98 \pm 1.06\) |

**TABLE I.** Best fit parameters for the \(T^4 \times Z_3\) model with \(\tan \beta = 2\). The minimum \(\chi^2 = 2.77\); here a coefficient of magnitude 3 or \(1/3\) contributes one unit to \(\chi^2\).

<table>
<thead>
<tr>
<th>Observable</th>
<th>Expt. value</th>
<th>Fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_t)</td>
<td>((3.3 \pm 1.8) \times 10^{-3})</td>
<td>(3.5 \times 10^{-3})</td>
</tr>
<tr>
<td>(m_d)</td>
<td>((6.0 \pm 3.0) \times 10^{-3})</td>
<td>(4.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(m_b)</td>
<td>(0.155 \pm 0.055)</td>
<td>0.136</td>
</tr>
<tr>
<td>(m_c)</td>
<td>(1.25 \pm 0.15)</td>
<td>1.24</td>
</tr>
<tr>
<td>(m_s)</td>
<td>(4.25 \pm 0.15)</td>
<td>4.25</td>
</tr>
<tr>
<td>(m_u)</td>
<td>(173.8 \pm 5.2)</td>
<td>170.4</td>
</tr>
<tr>
<td>(m_e)</td>
<td>((5.11 \pm 1%) \times 10^{-4})</td>
<td>(5.11 \times 10^{-4})</td>
</tr>
<tr>
<td>(m_\mu)</td>
<td>(0.106 \pm 1%)</td>
<td>0.106</td>
</tr>
<tr>
<td>(m_\tau)</td>
<td>(1.78 \pm 1%)</td>
<td>1.78</td>
</tr>
<tr>
<td>(</td>
<td>V_{us}</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>V_{ub}</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>)</td>
</tr>
</tbody>
</table>

| \(\Delta m_{23}^2 / \Delta m_{12}^2\) | \(100 - 2500\) | 526 |
| \(\ln (\Delta m_{23}^2 / \Delta m_{12}^2)\) | 6.22 \pm 1.61 | 6.27 |
| \(\sin^2 \theta_{12}\) | \(2 \times 10^{-3} - 10^{-2}\) | \(4.5 \times 10^{-3}\) |
| \(\ln (\sin^2 \theta_{12})\) | \(-5.41 \pm 0.80\) | \(-5.40\) |
| \(\sin^2 \theta_{23}\) | \(> 0.8\) | 0.90 |
| \(\sin^2 \theta_{13}\) | | \(1.4 \times 10^{-3}\) |

**TABLE II.** Experimental values versus fit central values for observables using the inputs of Table I. Masses are in GeV and all other quantities are dimensionless. Error bars indicate the larger of experimental or 1% theoretical uncertainties, as described in the text.

**SMA Solution**
### TABLE I.
Best fit parameters for the $T' \times Z_3$ model with $\tan \beta = 2$. The minimum $\chi^2 = 11.8$; here a coefficient of magnitude 2 or 1/2 contributes one unit to $\chi^2$.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Expt. value</th>
<th>Fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_u$</td>
<td>$(3.3 \pm 1.8) \times 10^{-3}$</td>
<td>$3.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$m_d$</td>
<td>$(6.0 \pm 3.0) \times 10^{-3}$</td>
<td>$4.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$m_s$</td>
<td>$0.155 \pm 0.055$</td>
<td>0.134</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$1.25 \pm 0.15$</td>
<td>1.20</td>
</tr>
<tr>
<td>$m_b$</td>
<td>$4.25 \pm 0.15$</td>
<td>4.24</td>
</tr>
<tr>
<td>$m_t$</td>
<td>$173.8 \pm 5.2$</td>
<td>168.0</td>
</tr>
<tr>
<td>$m_e$</td>
<td>$(5.11 \pm 1%) \times 10^{-4}$</td>
<td>$5.11 \times 10^{-4}$</td>
</tr>
<tr>
<td>$m_\mu$</td>
<td>$0.106 \pm 1%$</td>
<td>0.106</td>
</tr>
<tr>
<td>$m_\tau$</td>
<td>$1.78 \pm 1%$</td>
<td>1.78</td>
</tr>
<tr>
<td>$</td>
<td>V_{us}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>$</td>
</tr>
</tbody>
</table>

| $\Delta m_{23}^2/\Delta m_{12}^2$ | 20 - 1670 | 376 |
| $\ln (\Delta m_{23}^2/\Delta m_{12}^2)$ | $5.21 \pm 2.21$ | 5.93 |
| $\sin^2 2\theta_{12}$ | $> 0.8$ | 0.88 |
| $\sin^2 2\theta_{23}$ | $> 0.8$ | 0.83 |
| $\sin^2 2\theta_{13}$ | $< 0.18$ | 0.10 |

### TABLE II.
Experimental values versus fit central values for observables using the inputs of Table I. Masses are in GeV and all other quantities are dimensionless. Error bars indicate the larger of experimental or 1% theoretical uncertainties, as described in the text.

**LMA Solution**
Conclusions

1) Gauged discrete symmetries are viable candidates for flavor physics

2) Their additional small representations provide for novel solutions not available for continuous groups

3) It does not require a large group to explain successfully a great deal of flavor physics