From CKM Matrix to MNS Matrix:
A Model Based on Supersymmetric
SO(10) x U(2)_F Symmetry

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Introduction

- Super-K atmospheric $\nu$-data: $\nu_\mu \rightarrow \nu_\tau$

$$\Delta M_{\text{atm}}^2 = \Delta M_{13}^2 \simeq 10^{-3} - 10^{-2} \text{ ev}^2$$

$$\sin^2 2\Theta_{\text{atm}} = 0.88 - 1$$

- Solar $\nu$ experiments: $\nu_e \rightarrow \nu_\mu$


\begin{align*}
(\text{VO}) & \quad \Delta m_0^2 = \Delta m_{12}^2 \simeq 10^{-5} - 10^{-4} \text{ ev}^2, \quad \sin^2 2\Theta_0 \simeq 0.67 - 1 \\
(\text{LAMS}^*\text{W}) & \quad \Delta m_0^2 \simeq 10^{-5} - 10^{-4} \text{ ev}^2, \quad \sin^2 2\Theta_0 \sim 1 \\
(\text{LOW}^*\text{W}) & \quad \Delta m_0^2 \sim 10^{-4} \text{ ev}^2, \quad \sin^2 2\Theta_0 \sim 1 \\
(\text{SAMSW}) & \quad \Delta m_0^2 \sim 10^{-6} - 10^{-5} \text{ ev}^2, \quad \sin^2 2\Theta_0 \simeq 10^{-3} - 10^{-2}
\end{align*}

$\Rightarrow \; m_\nu \neq 0$

$\Rightarrow$ strong supports to SO(10).

- SUSY SO(10):

  - all 16 known fermions (including $\nu_e$) in each family are in one single 16-dim spinor representation of SO(10).
- Yukawa couplings: $16_i \otimes 16_j = 10 \oplus 120_A \oplus 126_s$

symmetric mass matrices: $10$, $\overline{126}_s$

**LR symmetry breaking chain:** $SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R$ $\rightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

\[ M_u = Y_{ij}^{10} \langle 10^+ \rangle + Y_{ij}^{126} \langle \overline{126}^+ \rangle \]

\[ M_d = Y_{ij}^{10} \langle 10^- \rangle + Y_{ij}^{126} \langle \overline{126}^- \rangle \]

\[ M_e = Y_{ij}^{10} \langle 10^- \rangle - 3 Y_{ij}^{126} \langle \overline{126}^- \rangle \]

\[ M_{\nu, LR} = Y_{ij}^{10} \langle 10^+ \rangle - 3 Y_{ij}^{126} \langle \overline{126}^+ \rangle \]

- up-quark $\leftarrow$ Dirac neutrino
- down-quark $\leftarrow$ charged lepton

- automatic $R_p$ symmetry:

\{ congruence number $0$: 45, 54, 210, ...

$2$: 10, 126, $\overline{126}$ ...

$\Rightarrow$ 16 and $\overline{16}$ are NOT in these classes.

- Majorana mass term:

\[ M_{\nu, RR} = Y_{ab}^{126} \langle \overline{126}^0 \rangle \]
Texture Analysis

- Symmetric, Real Mass Matrices with 5 texture zeros:

  RRR analyses: (Ramond, Roberts, Ross, 1993, NPB 406, 19)

  5 sets of up- and down quark texture combinations.

  with current experimental data, we found

  ⇒ only 1 set is viable (RRR texture vs).

- At GUT scale:

  \[ M_u = \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & 1 \end{pmatrix} \cdot d, \quad M_d = \begin{pmatrix} 0 & e & 0 \\ e & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot h \]

  \[ a \ll b \ll c \ll 1 \quad \quad e \ll f \ll 1 \]

- Georgi-Jarlskog relations:

  \[ m_b \approx m_\tau, \quad m_s = \frac{1}{3} m_\mu, \quad m_d \approx 3 m_e \]

  ⇒ \[ M_e = \begin{pmatrix} 0 & e & 0 \\ e & -3f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot h \]
• $M_{u}^{\text{diag}} = U_{u,l} M_{u} U_{u,r}^{+} = \text{diag} (m_{u}, m_{c}, m_{t})$
$$M_{D}^{\text{diag}} = U_{D,l} M_{D} U_{D,r}^{+} = \text{diag} (m_{d}, m_{s}, m_{b})$$
$$V_{\text{CKM}} = U_{u,l} U_{D,l}$$

• Neutrino Sector

- In $SO(10)$ at GUT scale: $M_{\nu,lr} \leftrightarrow M_{u}$

$$M_{\nu,lr} = \begin{pmatrix}
0 & 0 & \alpha \\
0 & \beta & \gamma \\
\alpha & \gamma & 1
\end{pmatrix} \eta, \quad \alpha = a, \beta = b, \gamma = c, \eta = d$$

- Seesaw Mechanism

$$\begin{pmatrix}
0 & M_{LR} \\
M_{LR}^{T} & M_{RR}
\end{pmatrix} \rightarrow \quad M_{\text{light}} = M_{LR}^{T} M_{RR}^{-1} M_{LR} \equiv M_{LL}$$
$$M_{\text{heavy}} \approx M_{RR}$$

- $M_{\nu,rr}$: in a "hidden" sector

$$M_{\nu,rr} = \begin{pmatrix}
0 & 0 & \delta_1 \\
0 & \delta_2 & \delta_3 \\
\delta_1 & \delta_3 & 1
\end{pmatrix} M_{R}$$

$$\delta_i = f_i (\alpha, \beta, \gamma, \eta)$$
- Form invariance seesaw:

\[ M_{\nu, LL} = M_{\nu, LR}^T M_{\nu, RR}^{-1} M_{\nu, LR} \]

\[ = \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 0 \\ t & 1 & 1 \end{pmatrix}, \quad t \ll 1. \]

- Neutrino Mixing:

\[ U_{\text{MNS}} = U_{e, LR}^T U_{\nu, LL} \sim U_{\nu, LL} \]

\[ = \begin{pmatrix}
-\frac{1}{\sqrt{2}} - \frac{t}{\sqrt{6}} \\
-\frac{1}{\sqrt{2}} + \frac{5t}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} - \frac{3t}{\sqrt{6}}
\end{pmatrix} \begin{pmatrix}
-\frac{1}{\sqrt{2}} - \frac{t}{\sqrt{6}} \\
-\frac{1}{2} + \frac{5t}{6} t^{1/2} \\
\frac{1}{2} + \frac{3t}{6} t^{1/2}
\end{pmatrix} \begin{pmatrix}
t/2 - 3t/16 \sqrt{3} \\
t/2 + 3t/16 \sqrt{3} \\
t/2 - 3t/16 \sqrt{3}
\end{pmatrix} \]

\[ |m_1| \approx \left( \frac{t}{\sqrt{6}} - \frac{t^2}{8} \right) \Lambda, \quad |m_2| \approx \left( \frac{t}{\sqrt{6}} + \frac{t^2}{8} \right) \Lambda, \quad |m_3| \approx \left( 2 + \frac{t^2}{4} \right) \Lambda \]

\[ \Delta m_{\text{atm}}^2 = |m_3|^2 - |m_2|^2 \sim O(1) \cdot \Lambda \]

\[ \Delta m_{\odot}^2 = |m_3|^2 - |m_1|^2 \sim O(10^3) \cdot \Lambda \]

\[ \Delta m_{\text{atm}}^2 \gg \Delta m_{\odot}^2 \]

\[ \Rightarrow \begin{cases} \text{hierarchical masses:} & |m_3| \gg |m_1|, |m_2| \\ \text{bimaximal mixing!} & \end{cases} \]
U(2) as a Flavour Symmetry

- **Froggatt-Nielsen Mechanism** ([Froggatt & Nielsen 1979, NPB47, 277])

  - **Tree level**
    \[ \phi_3 \rightarrow \psi_a \]
  
  - **Higher order**
    \[ \phi \rightarrow \phi_b \]

  \[ \langle H^0 \rangle \]

  - Introduce\( \{ \text{flavon fields} \phi \quad (\text{flavour Higgs fields}) \}
  \]
  
  - *vector-like matter fields* \( \tilde{x} + x \) (superheavy \( \sim M \))

  \[ W \sim \psi_a \left( 1 + \frac{\phi}{M} + \cdots \right) H \psi_b \]

- **Flavour Group Proposed:**
  - **Abelian** - non-abelian
  - **Global** - local
  - **Continuous** - discrete

- **Why U(2)?** ([Barbieri, Hall, Raby, Romanino, 1997, NPB493, 3])

  - Heaviness of top quark
  - SUSY FCNC
  - \( \mathcal{O}(1) \) coefficients determined. (Unlike U(1)_F case)

  \[ \psi_a \otimes \psi_3 = 2 \oplus 1, \quad a = 1, 2 \]

  \[ \psi_3 \otimes \psi_3 = 1, \quad \psi_a \psi_3 = 2, \quad \psi_a \psi_b = 3 \oplus 1 \]

  \( \Rightarrow \) relevant flavon fields

  \[ \phi^a \sim 2, \quad S^{ab} \sim 3, \quad A^{ab} \sim 1 \]
• $U(2)$ breaking in 2-steps:

$$U(2) \xrightarrow{EM} U(1) \xrightarrow{e'M} \text{nothing}$$

where

$$e, e' \sim \frac{\langle \text{flavon} \rangle}{M}, \quad M: \text{UV cutoff}$$

$$e' \ll e \ll 1.$$ 

• Flavon VEVs: generically, in some chosen basis,

$$\frac{\langle \phi^a \rangle}{M} \sim (e e'), \quad \frac{\langle S^{ab} \rangle}{M} \sim (e' e e'), \quad \frac{\langle A^{ab} \rangle}{M} \sim (-e' 0 0)$$

using only $\phi^a$ and $S^{ab}$:

$$M \sim \mathcal{O} \begin{pmatrix} e' & e' & e' \\ e' & e & e \end{pmatrix}, \quad \text{hierarchy built-in}$$

• Combine $SO(6)$ with $U(2)$: a generic superpotential $W$:

$$W = \psi_3 \psi_3 H + \frac{1}{M} \psi_3 H \phi^a \psi_a + \frac{1}{M} \psi_a H S^{ab} \psi_b$$
A Realistic Model and its Predictions

<table>
<thead>
<tr>
<th>SM</th>
<th>MSSM</th>
<th>MSSM</th>
<th>SUSY</th>
<th>SUSY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\bar{e}$</td>
<td>$M_{susy}$</td>
<td>$\Lambda_R$</td>
<td>$\nu_R^+$</td>
<td>$M_{GUT}$</td>
</tr>
<tr>
<td>$(10^2 \text{ GeV})$</td>
<td>$\approx m_t^{top}$</td>
<td>$(10^2 \sim 10^4 \text{ GeV})$</td>
<td>$2 \times 10^6 \text{ GeV}$</td>
<td>$M$</td>
</tr>
<tr>
<td>(175 GeV)</td>
<td>seesaw</td>
<td></td>
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in order to uniquely specify the superpotential without any unwanted terms $\Rightarrow \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ Symmetry

Field Content (Yukawa Sector)

| Matter: | $\Psi_a \sim (16, 2)^{+++}$, $\Psi_3 \sim (16, 1)^{+++}$ |
| Higgs:  | $(10, 1)$: $T_1^{+++}$, $T_2^{+++}$, $T_3^{+++}$, $T_4^{+++}$, $T_5^{+++}$ |
|         | $(126, 1)$: $\overline{C}^{---}$, $\overline{C}_1^{+++}$, $\overline{C}_2^{+++}$ |
| Flavon: | $(1, 2)$: $\Phi_1^{+++}$, $\Phi_2^{+++}$, $\Phi^{+++}$ |
|         | $(1, 3)$: $S_1^{+++}$, $S_2^{+++}$, $\Sigma^{+++}$ |

VEVs: $T_1: <10^+>, <10^->$

$T_{2,3,4}: <10_2^+, 10_3^+, 10_4^+>$

$T_5: <10_5^->$

$\overline{C}_{1,2}: <\overline{126}_1^>, \overline{C}: <\overline{126}^->$

$<10_1^+> = <10_3^+>$, $<10_1^-> = <10_5^->$, $<\overline{126}_1^> = <\overline{126}_2^->$
\[ \langle \Phi_1 \rangle = (e')_0, \quad \langle \Phi_2 \rangle = (e)_0, \quad \langle S_i \rangle = (e' e')_0, \quad \langle S_2 \rangle = (e' e')_0 \]
\[ \Phi = (\delta_1) \quad \Sigma = (0 \quad 0) \]

- **Superpotential:**

\[
W_{\text{Dirac}} = \psi_3 \psi_3 T_1 + \frac{1}{M} \psi_3 \psi_a (T_2 \Phi_1 + T_3 \Phi_2) \\
+ \frac{1}{M} \psi_a \psi_b (T_4 + \bar{c}) S_2 + \frac{1}{M} \psi_a \psi_b T_5 S_1 \\
W_{\nu_{\text{LR}}} = \psi_3 \psi \bar{c}_1 + \frac{1}{M} \psi_3 \psi_a \bar{c} \tilde{c}_2 + \frac{1}{M} \psi_a \psi_b \Sigma \bar{c}_2
\]

- **Mass matrices:**

\[
M_{\nu_{\text{LR}}} = \begin{pmatrix} 0 & 0 & \gamma_2 e' \\ 0 & \gamma_4 e & e \\ \gamma_2 e' & e & 1 \end{pmatrix} \langle 10^+ \rangle
\]

\[
M_{\nu_{\text{e}}} = \begin{pmatrix} 0 & e' & 0 \\ e' & (1, -3) p e & 0 \\ 0 & 0 & 1 \end{pmatrix} \langle 10^- \rangle
\]

\[
\gamma_3 \equiv \langle 10^+ \rangle / \langle 10^+_i \rangle, \quad \gamma_4 \equiv \langle 10^+_i \rangle / \langle 10^+_i \rangle, \quad \Phi \equiv \langle 126^- \rangle / \langle 10^- \rangle
\]

\[
M_{\nu_{\text{RR}}} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} \langle \overline{126}^{'},^0 \rangle
\]
TABLE I: Predictions and values extrapolated from experimental data. The first column shows the results calculated by Fusaoka et al [20]. The second column shows the Yukawa couplings at the GUT scale obtained with the input parameters we have chosen. The third column shows the predictions at $M_3$, after renormalization group effects have been taken into account.

\[
\begin{align*}
|V_{CKM,\text{prediction}}| &= \begin{pmatrix}
0.9751 & 0.2215 & 0.003541 \\
0.2215 & 0.9745 & 0.03695 \\
0.004735 & 0.03681 & 0.9993
\end{pmatrix} \\
|V_{CKM,\text{exp}}| &= \begin{pmatrix}
0.9751 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\
0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\
0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993
\end{pmatrix}
\end{align*}
\]

Neutrino Sector:

\[\delta_1 = 1.16 \times 10^{-3}, \quad \delta_2 = 3.22 \times 10^{-5}, \quad \delta_3 = 1.56 \times 10^{-7} \quad M_{R} = 1.32 \times 10^{14} \text{GeV} \]

\[m_1 = 2.0052 \times 10^{-4} \text{eV}, \quad m_2 = 2.0123 \times 10^{-4} \text{eV}, \quad m_3 = 0.05574 \text{eV} \]

\[\Delta m_{23}^2 = 3.11 \times 10^{-3} \text{eV}^2, \quad \Delta m_{12}^2 = 2.87 \times 10^{-10} \text{eV}^2 \]

\[|U_{MNS,\text{prediction}}| = |U_{eL}U_{\nu L}^\dagger| = \begin{pmatrix}
0.6710 & 0.7396 & 0.0527 \\
0.5410 & 0.4397 & 0.7169 \\
0.5070 & 0.5096 & 0.6952
\end{pmatrix}
\]

\[\sin^2 2\theta_{\text{atm}} \equiv 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = 0.9992 \]

\[\sin^2 2\theta_{\tilde{\alpha}} \equiv 4|U_{\tilde{\alpha} 3}|^2(1 - |U_{\tilde{\alpha} 3}|^2) = 0.9912 \]

\[m_1 \simeq 2.963 \times 10^7 \text{GeV}, \quad m_2 \simeq 2.643 \times 10^{10} \text{GeV}, \quad m_3 \simeq 1.319 \times 10^{14} \text{GeV} \]
It is possible to have the LAMSW solution with

$$\delta_1 = 0.001082, \quad \delta_2 = 0.0009870, \quad \delta_3 = 0.02238$$

$$M_R = 2.415 \times 10^{12} \text{GeV}$$

$$m_{\nu_1} = 0.01089 \text{eV}, \quad m_{\nu_2} = 0.01206 \text{eV}, \quad m_{\nu_3} = 0.09999 \text{eV}$$

and the squared mass differences are

$$\Delta m^2_{23} = 9.851 \times 10^{-3} \text{eV}^2, \quad \Delta m^2_{12} = 2.752 \times 10^{-5} \text{eV}^2$$

The lepton mixing matrix is given by

$$|U_{\text{MNS,prediction}}| = |U_{eL} U_{\nu L}^\dagger| = \begin{pmatrix}
0.6439 & 0.7486 & 0.1580 \\
0.6045 & 0.3712 & 0.7049 \\
0.4690 & 0.5494 & 0.6915
\end{pmatrix}$$

$$m_1 \simeq 5.732 \times 10^6 \text{GeV}, \quad m_2 \simeq 1.177 \times 10^9 \text{GeV}, \quad m_3 \simeq 2.417 \times 10^{12} \text{GeV}$$

(iii) LOW solution: $$\delta_1 = 1.15 \times 10^{-6}, \quad \delta_2 = 4.16 \times 10^{-6}, \quad \delta_3 \approx 1 \times 10^{-5}, \quad M_R \approx 1 \times 10^{13} \text{GeV}$$

$$m_{\nu_1} = 1.626 \times 10^{-3} \text{eV}, \quad m_{\nu_2} = 1.650 \times 10^{-3} \text{eV}, \quad m_{\nu_3} = 0.06303 \text{eV}$$

$$\Delta m^2_{23} = 3.973 \times 10^{-3} \text{eV}^2, \quad \Delta m^2_{12} = 1.248 \times 10^{-5} \text{eV}^2$$

$$|U_{\text{MNS}}| = \begin{pmatrix}
0.6665 & 0.7418 & 0.07428 \\
0.5511 & 0.4231 & 0.7192 \\
0.5021 & 0.5202 & 0.6909
\end{pmatrix}$$

**Summary**

- **mass hierarchy**: 2-step $U(2)$ breaking
- **SO(10) relations**: up-quark ↔ Dirac neutrino, down-quark ↔ charged lepton

7 + 4 parameters ⇒ predict $6 + 6 + 3 + 3 + (3)$

- $\nu$-masses
- mixing angles
- RH

- $R_p$ is preserved: $\overline{126}_H$