|Vcb| and |Vub|: Theoretical Status

- Introduction
  - why we care

- |Vcb|, \( \bar{\alpha} \) and \( \lambda_i \) (old stuff)
  - exclusive: \( \bar{B} \rightarrow D^{(*)} l\bar{\nu} \)
  - inclusive: \( \bar{B} \rightarrow X_{cl} l\bar{\nu} \), moments

- |Vub| (new stuff)
  - exclusive: \( B \rightarrow p (\pi) l\bar{\nu} \), wrong-sign charm, double ratios ...
  - inclusive: \( E_\ell \) vs. \( S_\ell \) vs. \( g^2 \) cuts

- Conclusions
For the purposes of this talk

MODEL DEPENDENT: theoretical
uncertainty is not parametrically
suppressed (in principle, $O(1)$)

MODEL INDEPENDENT: theoretical
uncertainty is parametrically suppressed
(typically $O\left( \frac{\text{ln} \; \text{cn}}{m^2} \right)^N$) (models
typically used to estimate size of this
uncertainty)
- the unitarity triangle provides a simple way to visualize SM relations...

\[
\begin{align*}
\frac{1}{\sin \theta_C} & \frac{|V_{ub}|}{|V_{cb}|} & \approx \frac{1}{\sin \theta_C} \frac{|V_{td}|}{|V_{ts}|}
\end{align*}
\]

\[\beta, \frac{|V_{td}|}{|V_{ts}|}, |V_{cb}|: \text{"easy" (theory & exp't both tractable)}\]

\[|V_{ub}|, \alpha, \delta: \text{Hard... want precision measurements to test CKM at \(~10\%\) level}\]
Illob: "easy", 2 complementary approaches (exclusive, inclusive) → current theoretical errors at ±5% level, well quantified

Illob: hard — no very good technique
- many approaches suggested
- current errors at ±20% level, NOT well quantified (spread in models)

THIS TALK: prospects with a $q^2$ cut
Vcb from Exclusive Decays: \( B \to D^{*0} \ell \bar{\nu} \)

- relies on \( m_b, m_c \gg \Lambda_{QCD} \)

\[
\frac{d\Gamma}{dw} (B \to D^{*0} \ell \bar{\nu}) = \frac{G_F^2 M_B^5 \Gamma^3}{48 \pi^3} \left( 1 - x^2 \right) \sqrt{w^2 - 1} (w + 1)^3
\]

\[
x \left[ 1 + \frac{4w}{1+w} \frac{1 - 2w^2 x + x^2}{(1-x^2)^2} \right] |V_{cb}|^2 f_{D^*}^2 (w)
\]

\[
\frac{d\Gamma}{dw} (B \to D \ell \bar{\nu}) = \frac{G_F^2 M_B^5 \Gamma^3}{48 \pi^3} \left( 1 - x^2 \right) (w^2 - 1)^{3/2} |V_{cb}|^2 f_D^2 (w)
\]

\[
\text{falls off faster at } \theta \text{ recoil}
\]

\[
\Gamma_\ell (w) \approx \frac{M_B^6 (w)}{M_B^4}
\]

\[
w = u \cdot v
\]

In \( m_b, c \to \infty \) limit, \( f_{D^*} (w) \approx f_D (w) \equiv \mathcal{F}(w) \) "square-wire function"

\[
f_{D^*} (1) = f_D (1) = 1 \quad \text{ABSOLUTELY NORMALIZED} \]
Including subleading corrections,

\[ F_+ (1) = \eta_A \left[ 1 + \frac{c}{m_c} + \frac{c}{m_b} + \delta \frac{1}{m^2} + \ldots \right] \]

2-loop radiative corrections

\[ \eta_A = 0.960^{+0.007}_{-0.007} \]

\[ \eta_A \] model dependence at sub-subleading order

\[ \delta \frac{1}{m^2} \approx -0.055 \pm 0.035 \]

\[ \Rightarrow F_+ (1) = 0.91 \pm 0.04 \] (BaBar book)

hard to improve upon w/o unquenched lattice

(final preliminary quenched: \( F_+ (1) = 0.93^{+0.03}_{-0.05} \))

CLEO (2003): \( |V_{cb}| = (46.4 \pm 2.0 \pm 2.1 \pm 2.1) \times 10^{-3} \)

LEP (2003): \( |V_{cb}| = (38.4 \pm 1.1 \pm 2.2 \pm 2.2) \times 10^{-3} \)

(using \( F_+ (1) = 0.88 \pm 0.05 \))
CLEO (2000): $v_{cb} = 0.0464 \pm 0.0020 \pm 0.0021 \pm 0.0021$

(preliminary)
$V_{cb}$ from inclusive decays: $\bar{B} \to X_c \ell \nu$

-only requires $m_b \gg \Lambda_{QCD}$ (not $m_c$)

\[ r \sim \frac{1}{m_b}: \text{b quark decays} \]

\[ r \sim \frac{1}{\Lambda_{QCD}}: \text{hadronization} \]

\[ \frac{d\Gamma}{d(p.s.)} \sim \text{parton model} + O(\frac{1}{m_b^2}) \]

"duality"

A bit of a cheat (need $m_b$ at leading order)
total inclusive width:

\[
\Gamma (\bar{B} \to X_c l\bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192 \pi^3} \times 0.369 \times \\
\left[ 1 - \frac{1.54 \, \alpha_s(m_b)}{\pi} - \left(1.43 \, \beta_0 + c\right) \left[\frac{d_s(m_b)}{\pi}\right]^2 \right],
\]

\[
\approx 10\%
\]

\[
\approx 6\%
\]

correlated (P.T. is actually better than it looks)

\[
-1.65 \, \frac{\lambda}{m_B} \left(1 - 0.87 \, \frac{\alpha_s}{\pi}\right) - 0.95 \, \frac{\lambda^2}{m_B^2}
\]

\[
- 3.78 \, \frac{d_1}{m_B^2} + 0.02 \, \frac{d_2}{m_B^2} + \ldots
\]

\[
\sim 3\%
\]

\[\lambda\]: mass of light d.o.f. in meson \(\sim o(1000)\)

\[-m^2 = \lambda_1\]: k.e. of heavy quark \(\sim o(1000)\)

\[-m^2 = \lambda_2\]: chromomagnetic interaction of heavy quark \(\sim o(1000)\)

\[-m^2 = 0.12\, \text{GeV}^2\]

\[m_B^2 = m_b + \lambda - \frac{d_1 + 3d_2}{m_b} + \ldots\] (NB: I have also treated me as heavy in the kinematics)

LEP: \(|V_{cb}| = (40.75 \pm 0.41 \, \text{exp} \pm 2.04 \, \text{th}) \times 10^{-3}\)
Other inclusive quantities (moments of spectra) are useful to

1) extract $\bar{t}, t$ to improve precision of $\bar{t}$, get quark masses (the same parameter occurs in expressions for different inclusive quantities)

2) check $1/m$ expansion, duality

Example:

- moments of $E_\ell$ (Voloshin)
- moments of $S_{K, M^2}$ (Falk, Hill, Savage)

$$R_1 = \frac{\int_{1.5 \text{ GeV}} E_\ell \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell}$$

$$R_2 = \frac{\int_{1.7 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{1.5 \text{ GeV}} \frac{d\Gamma}{dE_\ell} dE_\ell}$$

(not sensitive to model dependence of spectrum at low $E_\ell$)

(Gremm, Kapustin, Ligeti, Wise)
\[ \lambda_1 \text{ (GeV}^2) \]

\[ \Lambda \text{ (GeV)} \]

- \( \langle E_1 \rangle \text{ band} \)
- \( \langle (E_1 - \langle E_1 \rangle)^2 \rangle \text{ band} \)
- \( \langle M_x^2 - M_D^2 \rangle \text{ band} \)
- \( \langle (M_x^2 - M_D^2)^2 \rangle \text{ band} \)

(from CLEO-CONF-98-21)
COMMENTS:

1) Without \( \langle (s_{H-M_0}^2) \rangle \), bands are almost parallel in \((\tilde{\lambda}, \lambda_1)\) plane (same for \(R_1, R_2\))

\[ \Rightarrow \text{we want to measure } \langle E_y \rangle \text{ in } B \to X_s Y \]

(Kapustin, Ligeti, Bauer)

2) Second moment \( \langle (s_{H-M_0}^2) \rangle \) is not reliable \( \Rightarrow \text{convergence of } \frac{1}{m_q} \)

expansion is poor / nonexistent.

\[ \ln \left( \frac{\langle B \to X_s Y \rangle}{m^3} \right) \sim 1 - 0.13 \frac{\bar{\lambda}}{400 \text{MeV}} - 0.02 \frac{\lambda_1}{(400 \text{MeV})^2} - 0.003 \frac{P_1}{(400 \text{MeV})^3} + \ldots \]

\[ \langle (s_{H-M_0}^2) \rangle \sim 0.016 \frac{\bar{\lambda}}{400 \text{MeV}} + 0.008 \frac{\lambda_1}{(400 \text{MeV})^2} + 0.002 \frac{P_1}{(400 \text{MeV})^3} + \ldots \]

\[ K \langle (s_{H-M_0}^2) \rangle \sim 0 \times \bar{\lambda} - 6.8 \times 10^{-3} \frac{\lambda_1}{(400 \text{MeV})^2} - 5.1 \times 10^{-3} \frac{P_1}{(400 \text{MeV})^3} + \ldots \]

\[ \text{no convergence} \]

\( (P_1 \sim (500 \text{MeV})^3 \text{ from vacuum insertion}) \)
Estimated theoretical uncertainties in $(\bar{\Lambda}, \lambda_1)$

extracted from

--- $B \rightarrow X_s \gamma$, first moment of $F_X$ spectrum

--- --- $B \rightarrow X_c \ell \bar{\nu}$, first moment of $dX$ spectrum

(from C. Bauer, 19057-5811 (1993))
Some methods I will not discuss in detail:
(Exclusive semileptonic or nonleptonic)

\[ B \to p \ell \nu, \]
\[ \pi \ell \nu \]

CLEO '99: \(|V_{ub}| = (3.23 \pm 0.24 \pm 0.58) \times 10^{-3}\)
but form factors must be calculated from models (light cone sum rules, quenched lattice, quark models)
- no zero-recoil simplification in HQET
- theoretical error hard to quantify (average of models) & hard to improve
- with unquenched lattice, measurement at large \( q^2 \) will eventually win

Double ratios
\[ \frac{B \to p \ell \nu}{B \to K \ell \nu} \times \frac{D \to K^* \ell \nu}{D \to p \ell \nu} \]
(Higuti, Stewart, Wise)

"Wrong-sign charm"

\[ b \to u \ell \nu \]

"Wrong-sign charm"

Inclusive \((b \to u \ell \nu)\): theoretically pristine, but measurable?
(Falk & Petrou; Chay, Falk, ML & Petrou)

Exclusive \((B \to D_{s}^{+} e^{+} e^{-})\): calculable in heavy quark limit, but branch
(Gronstein, Nakao, Pijdo) \(~10^{-12}\)
In principle, as straightforward as the inclusive determination of $|V_{ub}|$:

$$\Gamma(\bar{B}\rightarrow X_u l\nu) = \frac{G_F^2|V_{ub}|^2 M_{B_s}^5}{192 \pi^3} \left[ 1 + O(\alpha_s) + O(\frac{1}{M_B^2}) + \ldots \right]$$

need $m_B$ (very sensitive)

but swamped by $\sim 100\times$ charm background

so must cut on appropriate kinematic variables to eliminate charm background

<table>
<thead>
<tr>
<th>CUT</th>
<th>EVENTS REMAINING</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) lepton energy $E_l &gt; \frac{M_B^2 - M_{B_s}^2}{2M_B}$</td>
<td>$\sim 10%$</td>
<td>$</td>
</tr>
<tr>
<td>2) hadronic invariant mass $S_H &lt; M_B^2$</td>
<td>$\sim 80%$</td>
<td>$</td>
</tr>
<tr>
<td>3) lepton invariant mass $q^2 &gt; (M_B - M_{B_s})^2$</td>
<td>$\sim 20%$</td>
<td></td>
</tr>
</tbody>
</table>

(Barger, Kim & Phillips; Bigi, Dikeman & Uraltsev; Falk, Hagedorn; Wise)

(Bauer, Hagedorn; ML)
\[ E_L \geq \frac{M_B^2 - M_0^2}{2M_B} \]
$S_H > M_D^2$
DANGEROUS REGION
(High E, low $g^2$)
over most of phase space, non-perturbative effects are

1. small \( \mathcal{O}\left(\frac{1}{M_{Pl}^n}\right) \)

2. parameterized by a few parameters at each order

\[ \ln : \text{energy of light d.o.f.} \]
\[ \lambda_1 : \langle E_{\text{kin}} \rangle \quad \text{etc.} \]
\[ \Rightarrow \#'s, \text{ not functions} \]

But in the low \( g^2 \), high \( E_{Pl} \) region,

things get more subtle...
Complications at high $E_L$, low $q^2$:

$P_b = m_b v + k$ of $b$ quark in hadron $\sim O(\lambda c o)$

$V^\mu = (1, 0, 0)$

$q \quad \text{leptons}$

$\Rightarrow$ decays

$P_x \quad \text{light hadrons}$

$S_H = P_x^2 = (m_b v - q)^2 + 2k \cdot (m_b v - q) + \ldots$

$O(m_b^2)$ over most of phase space

$O(\lambda c o, m_b)$ is suppressed over most of phase space

but at endpoint, $q = \left(\frac{m_b}{2}, 0, 0, \frac{m_b}{2}\right)$

$\Rightarrow m_b v - q = m_b (1, 0, 0, -1)$

$\Rightarrow (m_b v - q)^2 = 0$

$\Rightarrow$ at endpoint,

$S_H = m_b k^+ + \ldots$

$k^+ = k^0 + k^3$
Lesson:

- In high $E_t$, low $q^2$ region, usual description of inclusive $B$ decay (parton model + $O(C/mb^4)$ corrections) breaks down

- Light-cone wave function $f(k^+)$ of $b$ quark in meson determines shape of $S_H$ spectrum up to $S_H \sim \Lambda_{QCD} M_b$

  (NB: this is $\gg \Lambda_{QCD}$, the size of the resonance regime $\Rightarrow$ decay is still inclusive)

  Rate in this region is highly MODEL DEPENDENT

- At leading order in $1/M_b$, this may be taken into account by convoluting parton-level rate evaluated with an effective mass $\tilde{m} = m_b + k^+$ with $f(k^+)$:

  $$d\Gamma \sim \int dk^+ f(k^+) d\Gamma (M_b \rightarrow m_b + k^+)$$

  - Corresponds to resumming the most singular terms in the $1/M_b$ expansion

  (Nener, T.; Bg, Shifman, Udalov, Vainshtein)
Qu: what does this mean for \(|V_{ub}|\) extraction?

1. \(E_e > \frac{M_B^2 - M_0^2}{2M_B} \Rightarrow \Delta E_e \sim \frac{M_0^2}{2M_B}\)

   \(f(k_+)^v\) determines rate when \(\Delta E_e \leq \Lambda_{QCD}\)

2. \(S_H < M_0^2\), but \(f(k_+)^v\) determines rate

   for \(S_H \leq \Lambda_{QCD} M_B\)

So both extractions only model-independent

- if \(M_0^2 \gg \Lambda_{QCD} M_B\)

  ... unfortunately, \(M_0^2 \sim \Lambda_{QCD} M_B\)

  \(\therefore\) extraction of \(|V_{ub}|\) using either approach is model-dependent (requires unknown wave function \(f(k_+)\))

(\(NB: S_H\) extraction is less sensitive to "reasonable" variations in models - more reliable?)
\[ \frac{1}{\Gamma} \frac{d\Gamma}{dE_l} \quad (\text{GeV}^{-1}) \]

lepton energy spectrum

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dm_X^2} \quad (\text{GeV}^{-2}) \]

hadronic invariant mass spectrum

\[ \rho \text{ threshold} \]

\[ m_X^2 (\text{GeV}^2) \]
Solutions

1. $f(k^+)$ may be measured in $\Sigma$ spectrum of $B \to X s \Sigma^+$, and used to extract $|V_{ub}|$ via $E_d$ or $S_W$ cut (Neubert, Korchemskiy & S扪an, Akhoury & Rothstein, Leibovich, Woa & Rothstein).

   Limiting factor: unknown $\mathcal{O}(\Lambda_{QCD}/M_b)$ corrections (subleading shape $f(x)$).

2. Avoid dangerous region altogether → eliminate charm background with lepton $q^2$ cut (Bauer, Ligeti).

Advantages:
- No dependence on $f(k^+)$
- Non-perturbative corrections known to $\mathcal{O}(\Lambda_{QCD}/M_b)^3$.*
- Perturbation theory well behaved (no Sudakovs)
- Charm background suppressed at edge of phase space

Disadvantages:
- Experimentally feasible?
- Only ~20% of events (duality problems?)
- Nonperturbative corrections enhanced due to restricted phase space... really $\mathcal{O}(\Lambda_{QCD}/m_c)^3$ at best
\[ \frac{d\Gamma}{dq^2} \quad (B \to X_u l\bar{s}) \]

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dq^2} \quad (GeV^2) \]

\[ q^2 (GeV^2) \]

--- parton level
--- including shape function
Fraction of events with $q^2 > q_0^2$

$F(q_0^2)$

$\begin{align*}
q_0^2 (GeV^2) & \\
10 & \quad 12 & \quad 14 & \quad 16 & \quad 18 & \quad 20
\end{align*}$

uncertainty (%)

$\begin{align*}
q_0^2 (GeV^2) & \\
11 & \quad 12 & \quad 13 & \quad 14 & \quad 15 & \quad 16 & \quad 17 & \quad 18
\end{align*}$

est. uncertainty due to $1/m_b^2$ operators
Charm background to $\frac{d\Gamma}{dq^2} (B \rightarrow X u l \bar{v})$
**Est. Uncertainties:**

<table>
<thead>
<tr>
<th>$(q_0^2)$</th>
<th>$F'(q_0^2)$</th>
<th>$\delta m_b$</th>
<th>$\bar{\alpha}_s$</th>
<th>$\bar{\alpha}_s(N/M_c)$</th>
<th>$C'(\mu_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(M_b - m_0)^2$</td>
<td>$0.204 \pm 0.040$</td>
<td>16.7%</td>
<td>6%</td>
<td>3%</td>
<td>8.2%</td>
</tr>
<tr>
<td>$13\text{ GeV}^2$</td>
<td>$0.151 \pm 0.036$</td>
<td>18.5%</td>
<td>7.2%</td>
<td>4.7%</td>
<td>12.5%</td>
</tr>
<tr>
<td>$15\text{ GeV}^2$</td>
<td>$0.090 \pm 0.032$</td>
<td>22.2%</td>
<td>10%</td>
<td>9.8%</td>
<td>23.8%</td>
</tr>
</tbody>
</table>

(from M. Newbert, hep-ph/0006068)
(assume $\delta m_b = 80\text{ MeV}$)

Where \( |V_{ub}| = 2.96 \times 10^{-3} \left[ \frac{\text{Br}(B \to X d \ell \nu)}{10^{-2} F'(q_0^2)} \right]^{1/2} \cdot \frac{1.6 \text{ ps}}{2 \beta} \)
Conclusions:

\[ V_{cb} : \]

**Exclusive:** stuck with model dependence at $\mathcal{O}(\frac{1}{M_{c}^{2}})$

\[ \rightarrow \pm 5\% \text{ theoretical error} \]

\[ \rightarrow \text{hard to see improvement w/o unquenched lattice} \]

**Inclusive:** theoretical error in principle smaller

\[ \rightarrow \text{need } \frac{m_{b}}{\Lambda} : \pm 50 \text{ MeV determination} \]

\[ \text{of } \Lambda \rightarrow \pm 2\% \text{ in rate} \]

Checks:

- exclusive form factor ratios
- moments (including $<E_{y}>$ in $B \to K_{s} \ell^{+} \ell^{-}$)

\[ V_{ub} : \]

current methods $\rightarrow$ large model dependence

- can eliminate via $\mathcal{O}$ comparison with $B \to K_{s} \ell^{+} \ell^{-}$, or $\mathcal{O} q^{2}$ cut to eliminate charm background

- a model-independent, 10% extraction does not seem unreasonable...

- want to try as many methods as possible...