Glueballs on a Transverse Lattice

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- hep-lat/991105
- Transverse Lattice formulation of gauge theory
- Computational strategy
- Some results
- What's next
Transverse Lattice formulation of gauge theory

"time" \( x^+ = \frac{x^0 + x^3}{\sqrt{2}} \)

"longitudinal" \( x^- = \frac{x^0 - x^3}{\sqrt{2}} \)

\( x^1, x^2 \): lattice with spacing \( a \)

\( A^+ A^- \) live on lattice sites
\( M \) lives on lattice links
\( N \times N \) complex matrix
* Computational strategy

1. Write down Hamiltonian $P$ consistent with:
   * gauge invariance
   * remaining Poincaré symmetries
   * locality (also parity)
   * few particle interactions (<4)

2. Determine unknown coupling constants (9)

Demand Lorentz invariance of low-lying observables to determine couplings

• Uniquely QCD since $x^+ + x^-$ directions are continuum QCD
• Have more observables than couplings (>15)
Unique Lorentz-invariant
Scaling trajectory

\[ H \text{ infinite-dimensional space of coupling constants} \]

Approximate trajectory

\[ H_s \text{ space (9 dimensions)} \]
Some results

1. generate and diagonalise Hamiltonian matrix on computer

2. calculate:
   - glueball spectrum
   - "winding modes"
   - heavy source potential
   - non-zero momentum states $p, p^* \neq 0$

3. demand
   - correct relativistic dispersion of spectra
   - isotropy of various measurements of string tension
   - degeneracy of spin multiplets

4. Find a unique path in parameter space where criteria are best satisfied.
Invariance of spectrum along trajectory $T_s$

\[ \frac{m}{\sqrt{\sigma}} \]

0$^{++}$

\[ \frac{M}{\sqrt{\sigma}} \]

2$^{++}$

\{ 0$^+$, 1$^+$, 2$^-$ \}

\[ \frac{M}{\sqrt{\sigma}} \]

1$^+$

\{ 0$^-$, 1$^-$ \}

\[ \mathcal{J}_3 \]
Figure 1: The variation of glueball masses with $N$ (pure glue). ELMC predictions are continuum ones for $N = 2, 3$ [19, 18, 20] and fixed lattice spacing estimates for $N = 4$ [35]. The dotted lines are to guide the eye and correspond to leading linear dependence on $1/N^2$. 
Gluon distribution Functions

\[ J^{\rho c} = 0^{++} \]

\[ 2^{++} \]

\[ 1^{+-} \]
Conclusion

The transverse lattice provides:
- Independent first principles predictions
- Wave functions
- Confirmation of large \( N_c \) expectations

Future work includes:
- Include fermions
- Finite \( N \)
- Heavy quark potential (numerically more difficult)
- Understand connection to the continuum theory (zero modes)
- Hadronization (jet production)