AN INVESTIGATION OF THE ASYMMETRIC O(ε) IMPROVED FERMILAB LATTICE ACTION FOR HEAVY QUARKS

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ASYMMETRIC O(α) IMPROVED FNAL
LATTICE ACTION FOR HEAVY QUARKS

IMPLEMENTATION OF IDEAS DEVELOPED IN
A.X. EL-KHADRA, A.S. KRONFELD, P.B. MACKENZIE
"MASSIVE FERMIONS IN LATTICE GAUGE THEORY"
PHYS. REV. D 55 (1997) 3983

SYMANZIK IMPROVEMENT OF THE WILSON
ACTION WITH AN INTERPRETATION INFORMED
BY HQET EXPANSION -
LEADS TO FORMULATION WITHOUT FULL
LORENTZ SYMMETRY OF OPERATORS
AND WITH FULL MASS DEPENDENCE OF
COEFFICIENTS.
SMOOTH CONNECTION BETWEEN LIGHT QUARK
(S.W. ACTION) AND HEAVY QUARK (NRQCD)
LIMITS.
HQET ANALYSIS $\Rightarrow$ TWO MASS TERMS IN ACTION

REST MASS

KINETIC MASS

SYMMETRIC (WILSON) ACTION $\Rightarrow O(qM^2)$ ERROR
$\Rightarrow$ REST MASS $\neq$ KINETIC MASS

FIX THIS WITH SPACE-TIME ASYMMETRY

HQET ANALYSIS $\Rightarrow$ REST MASS IRRELEVANT

$\therefore$ CAN USE SYMMETRIC ACTION IF WE USE KINETIC MASS TO FIX THE PHYSICS

TEST THIS.
LATTICE DISPERSION RELATION:

\[ E^2(p^2) = M_1^2 + \frac{M_1}{M_2} p^2 + O(p^4) \]

Define rest mass \( M_1 = E(0) \).
Kinetic mass \( M_2 = \left( \frac{\partial^2 E}{\partial p^2} \right)_{p=0}^{-1} \).

\( M_1 \neq M_2 \) to \( O(\alpha M_0) \).

Correct for this by introducing a space-time asymmetry in the fermion action.

Adjust the asymmetry until \( M_1 = M_2 \).

Constitutes the first improvement condition.

\( M_2 \) is the physically relevant mass.
LATTICE ACTION

\[ S_0 = \bar{\psi} \psi - K_t \bar{\psi} D_t \psi \]

\[- K_s \sum \bar{\psi} D_i \psi \]

ASYMMETRY PARAMETER \[ J = \frac{K_s}{K_t} \]

BARE QUARK MASS \[ M_0 = \frac{1}{2K_t} - 3J - 1 - M_{\text{crit}} \]

AT SOME \[ J = J_{\text{NP}} (M_0) \quad M_1 = M_2 \]

O(a) IMPROVEMENT:

\[ S_E = C_E \bar{\psi} iK_s \sigma_0 F_0 \]

\[ S_B = C_B \bar{\psi} iK_s \sigma_{ij} F_{ij} \]

O(a) IMPROVED ASYMMETRIC ACTION

\[ S = S_0 + S_E + S_B \]
LATTICE DETAILS:

$12^3 \times 24$

$\beta = 5.7$

QUENCHED

$c_\varepsilon = c_8 = 1.57$ (TADPOLE IMPROVED PERT. THEORY)

COMPARE WITH

PERT. THEORY

MERTENS, KRONFELD, EL-KHADRA

"SELF ENERGY OF MASSIVE LATTICE FERMIONS"

PHYS. REV. D58 (1998) 034505

$J=1$ M.C. DATA (J. SIMONE)

ADDITIONAL LATTICE RESULTS:

$16^3 \times 32$

$\beta = 5.9$

QUENCHED

$c_\varepsilon = c_8 = 1.50$
STRATEGY FOR FINDING $\lambda_{np}$

Compute $M_1$ and $M_2$ for $1S$ state of quarkonium and heavy-light mesons

(light quark at strange mass computed with $J = 1$)

Adjust $J$ until $\frac{M_1}{M_2} = 1$

Do this at constant $M_0$
To get mass dependence

Or at constant $M_2$ (const. $k_3$)
To get physics

Quarkonium: 100 configs
300 configs at $b$ and $c$

Heavy-lights: 300 configs
(larger statistical errors)
Calculation of $m_1$ and $m_2$:

Compute $p_5$ and $v$ correlation functions at 5 momenta using Coulomb gauge 1S and 2S hydrogenic wavefunction sources and sinks.

Perform 2 state fit to matrix of correlators

\[
\begin{pmatrix}
1S-1S & 2S-1S \\
1S-2S & 2S-2S
\end{pmatrix}
\]

Obtain $E(p)$

Then $m_1 = E(0)$

Extract $m_2$ from $a_1$ in fit of dispersion relation to

\[
E^2(p^2) = a_0 + a_1 p^2 + a_2 p^4 + a_3 \sum p_i^4
\]
INTERPOLATION TO $J_{np}$ IN QUARKONIUM
AT CONSTANT $\alpha M_0 = 0.72$
\[ \zeta_{\text{NP}} \text{ HH } \beta = 5.9 \]

\[ \zeta_{\text{NP}} \text{ HL } \beta = 5.9 \]
$J^{NP}$ as a function of quark mass
from quarkonia and heavy-lights
QUARKONIUM DISCREPANCY

\[ J_{np} \text{ from quarkonia does not match that from heavy-lights} \]
\[ J_{np} \text{ from heavy-lights agrees reasonably with p.t.} \]

FOLLOWING SCRIP

COLLINS, EDWARDS, HELLER, SLOAN

NUCL. PHYS. (PROC. SUPPL.) B47 (1996) 485

DEFINE

\[ I = \frac{2 \delta M_{nl} - (\delta M_{hh} + \delta M_{ll})}{2M_{2, nl}} \]

\[ \delta M = M_2 - M_1 \]

PLOT \( I \) vs. \( M_{2, nl} \) with the heavy quarks computed at \( J_{np} \) from the heavy-lights, so \( \delta M_{nl} = 0 \) (and \( \delta M_{ll} = 0 \))
This effect explained in

A. S. Kronfeld


In terms of meson binding energies

\[ B_1 = M_{1qq} - M_{1q} - M_{1q} \]

\[ B_2 = M_{2qq} - M_{2q} - M_{2q} \]

For heavy-lights

Errors on \( B_1 \) and \( B_2 \) are \( O(\alpha_s, \alpha_s \Lambda_{QCD}) \)

with \( O(\alpha) \) improved action

For quarkonia

\( B_1 \) accurate to \( O(u^2) \)

\( B_2 \) accurate to \( O(1) \)

With action accurate to \( O(u^2) \)

Since \( M_2 \) multiplies an \( O(u^2) \) operator
$c\bar{c}$ Spectrum at Tuned $J$

$a^{-1} = 1.182^{+0.027}_{-0.028}$ GeV
$c\bar{c}$ Spectrum: Hyperfine + p-Wave Splitting

Charmonium $\bullet \; \xi$ tuned $\bullet \; \xi = 1$

$J/\psi - \eta_c$ 2S-1S $h_c - \chi_{c0}$ $h_c - \chi_{c0}$ $\chi_{c1} - \chi_{c0}$
$c\bar{c}$ SPECTRUM: SPLITTINGS FROM $1S$

$a_c' = 1.182^{+0.027}_{-0.028}$ GeV

$a_{c'} = 1.161^{+0.032}_{-0.032}$ GeV

Charmonium

- $\xi$ tuned
- $\xi = 1$
QUARKONIUM 1P-1S SPLITTING \ versus \ M_2

\[ aM(1P_1) - aM(1S) \]

- \( \xi_{NP} \)
- \( \xi = 1 \)
- \( \xi \neq 1 \)

\[ aM_2(1S) \]
HEAVY-LIGHT HYPERFINE SPLITTING
AS A FUNCTION OF $M_2$

\[ aM(\tilde{3}S_1) - aM(\tilde{1}S_0) \]

\[ aM_2(\tilde{1}S) \]
VARIATION OF THE PSEUDOSCALAR DECAY CONSTANT WITH $M_2$
CONCLUSIONS

DEMONSTRATED VIABILITY OF NON-PERTURBATIVE EVALUATION OF $J_{NP}$

REPRODUCTION OF PHYSICAL RESULTS - AGREEMENT WITH $J=1$ RESULTS

EVIDENCE FOR USING $M_2$ AS THE PHYSICALLY RELEVANT MASS.

POSSIBLE $O(\Lambda_{QCD}^2 \frac{a}{m_2})$ AND $O(m_2 v^2)$ ERRORS IN HEAVY-LIGHT AND QUARKONIA FROM USE OF TREE-LEVEL $C_6$ AND $C_8$

.: NEED COMPUTATION AT ANOTHER LATTICE SPACING

QUARKONIUM DISCREPANCY $\Rightarrow$ NEED TO FURTHER IMPROVE THE ACTION.