Neutrinos as Probes of Extra Dimensions

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* Introduction
* Large extra dimensions
* Warped extra dimensions
* Summary


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Introduction

Conventional view:

- neutrinos probe short-distance physics scale $M \leq M_{\text{GUT}} \sim 10^{16}$ GeV
- RHD neutrino = SM singlet, not protected from acquiring large mass
  $\Rightarrow m_\nu \sim \frac{v^2}{M}$ (see-saw)
- probe New Physics but in an indirect way (via coefficients of higher-dim. operators)

learn about scale of New Physics, but not its nature
Recently:

- revival of interest in models with extra (compact, spatial) dimensions
  \[ \Rightarrow \] provide new perspective (solution?) on hierarchy problem (why \( v \ll M_{pl} \)?)
  \[ \Rightarrow \] cosmological constant ... (?)

- RHD neutrino still special since it is a SM singlet and probes physics
  \& probes bulk geometry
  \[ \Rightarrow \] gravitational scales

BUT: these New Physics effects are now associated with the TeV scale?

\[ \Rightarrow \] can be probed directly at colliders
Neutrino physics in extra dimension models can be quite different from conventional picture:

* natural to have (infinitely) many sterile neutrinos
* natural to have light Dirac neutrinos without see-saw
* natural to have mixing matrix without hierarchy (unlike CKM)
* natural to have different numbers of LHD and RHD neutrinos

etc.
ADD: extra spatial dimensions could be macroscopic (≤ 1 mm) if all SM fields were confined to a 4D slice ("brane") of a higher-dim. space ("bulk")

- all particles carrying SM charges are confined to brane
- gravity lives in bulk

⇒ weakness of gravity is consequence of volume suppression
Planck scale is derived, effective 4D parameter:

\[ M_P^2 = M_*^{2+n} V_n \]

fundamental scale of gravity

Modification of Newton's law at scales \(< R_n\)

\[ R_n \leq 1 \text{ mm possible} \]

- Want \( M_* \sim \text{TeV} \) to eliminate hierarchy problem (gravity \( \sim \text{weak scale})

n = 1 excluded

n = 2: \( R_n \sim 1 \text{ mm} \)

n > 2: can have some \( R_n \) large
* sterile particle (SM singlet) is not confined to brane and thus lives in bulk:

\[ \text{RHD neutrino} \Leftrightarrow \text{bulk fermion} \]

(Arkani-Hamed et al.)

(Dienes et al.)

* consequences:

* coupling to SM neutrino is suppressed by same volume factor as gravit. coupling:

\[ m_{\nu} \sim v \cdot \frac{M^*}{M_p} \text{ like see-saw} \]

* bulk fermion has Kaluza-Klein excitations with spacings

\[ \Delta m \sim \frac{1}{R_n} \gtrsim 10^{-3} \text{ eV} \]

\[ \leftrightarrow \text{ mixing with SM neutrinos!} \]

* bulk neutrino more like graviton than like a SM fermion
Concrete model: (Dvali, Smirnov)

- Assume $M_a \sim 1$ TeV and $n \geq 2$ but one dimension (size $R$) much larger than others.

- Coupling of bulk states to SM neutrino gives Dirac mass term:

$$m_D = \frac{h v M_a}{M_p} \sim h \cdot 6 \cdot 10^{-5} \text{ eV}$$

$O(1)$ Yukawa coupling

- KK Dirac masses: $m_n = \frac{n}{R} \gg m_D$

$\Rightarrow$ diagonalize mass matrix

$$y_1 \sim 10^{-5} \text{ eV}$$

$$y_2 \sim 10^{-3} \text{ eV}$$

$$\vdots$$

$\Rightarrow$ predominantly sterile

$\Rightarrow$ predominantly $\nu_e$
\[ \nu_e \sim \nu_0 + \xi \sum_{n \geq 1} \frac{1}{n} \nu_n \]

coherent superposition of mass eigenstates with increasing mass and decreasing mixing, where

\[ \xi = \frac{\text{m}_\text{D}}{\text{m}_1} \ll 1 \]

* explain solar neutrinos in terms of \( \nu_e \to \nu_s \) where \( \nu_s \) is (predominantly) the first excited bulk mode:

\[ \Delta m^2 = \frac{1}{R^2} \sim (4 - 10) \times 10^{-6} \text{ eV}^2 \]

\[ \Rightarrow R \sim (0.06 - 0.1) \text{ mm} \]

\[ \sin^2 2\theta = 4 \xi^2 \sim (0.7 - 1.5) \times 10^{-3} \]

\[ \Rightarrow \text{requires small mixing angle solution!} \]

now strongly disfavored by Super-K
* resonance conversion in sun (MSW):

\[ E_1^R \sim 0.4 - 0.8 \text{ MeV} \]
\[ E_2^R \sim 1.6 - 3.2 \text{ MeV} \]
\[ E_3^R \sim 3.6 - 7.2 \text{ MeV} \]
\[ E_n^R = n^2 E_1^R \]

\[ \Rightarrow \text{characteristic modulation of survival probability as function of } E_v \]

\[ 4 E^2 = 10^{-3} \]
\[ \sin^2 2\theta = 10^{-3} \]
\[ \sin^2 2\theta = 4 \cdot 10^{-3} \]

conversion into bulk states
* limitations:

- difficult to explain more than one neutrino anomaly at same time
  
  (Mohapatra et al.)

- constraints from universality
  
  \[ \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} \]  
  (Das, Kong)
Warped extra dimensions

RS: extra dimension of few x Planck size (small), but non-factorizable geometry

\[ ds^2 = e^{-2k\,y_5} \eta_{\mu\nu} \, dx^\mu dx^\nu - dy_5^2 \]

Planck brane

our brane

\[ y_5 = 0 \quad \quad y_5 = \pi R_5 \]

sterile neutrinos

gravitons

\[ S'/Z_2 \]

compactification of 5th dim.

hierarchy explained by induced metric on our brane:

\[ g_{\mu\nu} \sim e^{-k_\pi R_5} = \frac{v}{M_{pl}} \sim 10^{-11} \]

if \( k R_5 \sim 12 \)

\[ H_{curvature} \]
no volume suppression for gravitons, so $M_* \sim M_p$ fundamental gravity scale

How to get small neutrino masses? (Grossman + MN)

* RS branes = domain walls and can support fermion zero modes

$\mathbb{Z}_2$ symmetry: - no LHD zero mode
- RHD zero mode localized on Planck brane?
Consequences:

* lightest state gets Dirac mass

\[ m_\nu \sim v \left( \frac{v}{M_p} \right)^{\frac{m_\nu - \frac{1}{2}}{k}} \]

\[ \text{parameter}! \]

\[ \Rightarrow \text{generalized see-saw relation} \]

\[ \Rightarrow \text{get } m_\nu \sim 10^{-5} - 10 \text{ eV for } \frac{m_\nu}{k} \sim 1.1 - 1.5 \]

* infinite tower of massive neutrinos with level spacing \( \Delta m \sim \text{TeV} \)

\[ v_3 \]
\[ v_2 \]
\[ v_1 \]
\[ v_0 \]

"anything" \( \sim \text{TeV} \)

\[ \left\{ \begin{array}{c}
\text{predominantly sterile} \\
\text{predominantly } v_e
\end{array} \right. \]
Minimal model: \((\text{Grossman + MN})\)

- 2 bulk fermions \(\rightarrow\) 2RHD zero modes

- yields:

\[
\begin{align*}
m_{\nu_1} &= 0 \\
\text{m}_{\nu_2} \text{ superlight} &\quad \Delta m_{12}^2 \sim v^2 \left( \frac{v}{M_p} \right)^2 \frac{m_{\nu_2}}{K} - 1 \\
\text{m}_{\nu_3} \text{ light} &\quad \Delta m_{23}^2 \sim v^2 \left( \frac{v}{M_p} \right)^2 \frac{m_{\nu_3}}{K} - 1
\end{align*}
\]

\[\Rightarrow\] naturally large hierarchy!

- mixing matrix: no small parameter

\[\Rightarrow\] no hierarchy (unlike CKM)!

- prefers LMA solar solution

- likes large atmospheric mixing

- naturally predicts large \((\text{max.} 1 \, \text{eV})\)

\[\Rightarrow\] great scenario for \(\nu\) factory!
Further predictions:

\[ \nu_e = \cos \theta_e \nu_e + \ldots \]

\[ \uparrow \]

light state

* invisible \( Z^0 \) width:

\[ 3 - N_v^{\text{eff}} = \sin^2 \theta_e + \sin^2 \theta_{\mu} + \sin^2 \theta_{\tau} \]

\[ = 0.015 \pm 0.008 \]

\[ \Rightarrow | \theta_i | \leq 0.1 \]

* universality: SM

\[ \frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} = 1.233 \cdot 10^{-4} \frac{\cos^2 \theta_e}{\cos^2 \theta_{\mu}} \]

\[ = (1.230 \pm 0.004) \cdot 10^{-4} \]

\[ \Rightarrow \text{similar bound} \]
theory:

\[ |\Theta_i| \sim 0.1 \text{ natural, but cannot be much smaller} \]

\[ \downarrow \]

expect to see deviations at level not much below current sensitivity?
Conclusions

* extra dimension models are full of surprises → shows our ignorance about physics beyond weak scale

* neutrino physics in extra dim. models can be very different from conventional lore

  RHD neutrino probes bulk geometry

* models make predictions that can be tested at ν factory and other collider experiments

  ↓

  go explore the "brane world" ...