Quintessence Model and Cosmic Microwave Background

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The Issue of Lambda

- S. Weinberg, RMP 61, 1 (1989)
- Y.J. Ng, IJMP D1, 145 (1992)

- The theoretical expectation for Lambda exceeds its observational limit by 120 orders of magnitude.

- In 1917 Einstein looked for a static solution of GR for cosmology.

- He found that by adding a lambda term such a static solution existed with density and mass depending on the non-zero parameter Lambda.
CBR TEMPERATURE ANISOTROPY

• The CBR was discovered in 1965 by Penzias and Wilson.

• But detection of its temperature anisotropy waited until 1992 when the Cosmic Background Explorer (COBE) satellite provided impressive further experimental support for the Big Bang model. The results are consistent with a scale-invariant spectrum of primordial scalar density fluctuations, such as might be generated by quantum fluctuations during inflation.
COBE's success

- Inspired many further experiments with smaller angle resolution than COBE (about 1 arc degree). NASA has approved a satellite mission MAP (Microwave Anisotropy Probe) for 2001. ESA has approved Planck surveyor - even more accurate than MAP - in 2006.

- With these experiments, the location of the first acoustic (Doppler) peak and possibly subsequent ones will be resolved.
The goal of the CBR experiments is to measure the temperature autocorrelation function. The fractional perturbation as a function of direction \( \hat{n} \) is expanded in spherical harmonics

\[
\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n})
\]

\[
(2l + 1)C_l = \sum_m a_{lm}
\]

A plot of \( C_l \) versus \( l \) will reflect oscillations in the baryon-photon fluid at the surface of last scatter. The first Doppler peak should be at \( l_1 = \pi/\Delta \Theta \) where \( \Delta \Theta \) is the angle subtended by the horizon at the time of last scatter (\( Z_{rec} \sim 1,100 \)).
The Special Case $\Lambda = 0$.
When $\Lambda = 0$, the Einstein-Friedman cosmological equations can be solved analytically (not generally true if $\Lambda \neq 0$).

We will find $l_1 \sim 1/\sqrt{\Omega_M}$.

Take

$$ds^2 = dt^2 - R^2 [d\Psi^2 + \sinh^2 \Psi d\Theta^2 + \sinh^2 \Psi \sin^2 \Theta d\Phi^2]$$

For a geodesic $ds^2 = 0$ and so

$$\frac{d\Psi}{dR} = \frac{1}{R}$$

Einstein's equation reads:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} + \frac{1}{R^2}$$

whence

$$\dot{R}^2 R^2 = R^2 + aR \quad a = \Omega_0 H_0^2 R_0^3$$

so that

$$\frac{d\Psi}{dR} = \frac{1}{\sqrt{R^2 + aR}}$$
\[
\Psi_t = \int_{R_0}^{R_t} \frac{dR}{\sqrt{(R + a/2)^2 - (a/2)^2}}
\]

Substitution of \(R = \frac{a}{2}(coshV - 1)\) gives:

\[
\Psi_t = cosh^{-1}\left(\frac{2R_0}{a} - 1\right) - cosh^{-1}\left(\frac{2R_t}{a} - 1\right)
\]

Noting that \(sinh(cosh^{-1}x) = \sqrt{x^2 - 1}\) gives for \(sinh\Psi_t\):

\[
\sqrt{\left(\frac{2(1 - \Omega_0)}{\Omega_0} + 1\right)^2 - 1} - \sqrt{\left(\frac{2(1 - \Omega_0)R_t}{\Omega_0 R_0} + 1\right)^2 - 1}
\]

Now

\[
\Delta \Theta = \frac{1}{H_t R_t sinh\Psi_t}
\]

For \(Z = 1, 100\) (recombination):

\[
l_1(\Lambda = 0) \simeq \frac{2\pi Z_t^{1/2}}{\sqrt{\Omega_M}} \simeq \frac{208}{\sqrt{\Omega_M}}
\]

See the figure on the next transparency:
Figure 1.
GENERAL CASE $\Lambda \neq 0$

In this case:

$$\dot{R}^2 R^2 = -k R^2 + a R + \Lambda R^4 / 3$$

Define

$$\Omega_M = \frac{8 \pi G \rho}{3 H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3 H_0^2}, \quad \Omega_C = \frac{-k}{H_0^2 R_0^2}$$

which satisfy

$$\Omega_M + \Omega_\Lambda + \Omega_C = 1$$

Then

$$l_1 = \pi H_t R_t sinh \Psi_t$$

where

$$\Psi_t = \sqrt{\Omega_C} \int_1^\infty \frac{dw}{\sqrt{\Omega_\Lambda + \Omega_C w^2 + \Omega_M w^3}}$$

After changes of variables one arrives at:
\[ l_1 = \pi \sqrt{m} \left( \frac{R_0}{R_t} \right) \times \]

\[ \times \sinh \left( \int_1^\infty \frac{dw}{\sqrt{\Omega_\Lambda + \Omega_C w^2 + \Omega_M w^3}} \right) \]

(for positive curvature $k = +1$ replace $\sinh$ by $\sin$)

For $\Omega_C = 0$ (inflation) this reduces to

\[ l_1 = \pi \sqrt{m} \left( \frac{R_0}{R_t} \right) \int_1^\infty \frac{dw}{\sqrt{\Omega_\Lambda + \Omega_M w^3}} \]

These are elliptic integrals, easily do-able with Mathematica. They resemble the well-known formula for the age of the universe:

\[ a = \frac{1}{H_0} \int_1^\infty \frac{dw}{w \sqrt{\Omega_\lambda + \Omega_C w^2 + \Omega_M w^3}} \]
We now plot data.

(II-13) \( l_1 \) vs. \( \Omega_M \) for \( \Omega_C = 0 \).

(II-14) iso- \( l_1 \) lines on a \( \Omega_M - \Omega_\Lambda \) plot for general \( \Lambda \) (the principal result of this analysis). The lines are for \( l_1 = 150 \rightarrow 270(R \rightarrow L) \) with \( \Delta L = 10 \).

(II-15) 3-dimensional rendition of \( l_1 \) vs. \( \Omega_M \) (right) and \( \Omega_\Lambda \) (front).
Now we look at the cumulative world data on $C(l)$ versus $l$.

- (II-17) shows current data. [With some license one may say that $l(1)$ is between 150 and 270 and hence within the bands of (II-14)]

- (II-18) shows quality of data to be expected from MAP and PLANCK experiments. Recall MAP is NASA 2000 and PLANCK is ESA 2005.
Figure 2: Current CMB data.
Figure 3: Simulated MAP and Planck data.
\[ \delta T = \left( \frac{(I+1)C_\ell}{2\pi} \right) \mu K \]

(Compiled by M. Tegmark, Jan. 00)

http://www.hep.upenn.edu/~mao/cmb/experiments.gif
A flat Universe from high-resolution maps of the cosmic microwave background radiation


The blackbody radiation left over from the Big Bang has been transformed by the expansion of the Universe into the nearly isotropic 2.73 K cosmic microwave background. Tiny inhomogeneities in the early Universe left their imprint on the microwave background in the form of small anisotropies in its temperature. These anisotropies contain information about basic cosmological parameters, particularly the total energy density and curvature of the Universe. Here we report the first images of resolved structure in the microwave background anisotropies over a significant part of the sky. Maps at four frequencies clearly distinguish the microwave background from foreground emission. We compute the angular power spectrum of the microwave background, and find a peak at Legendre multipole $l_{\text{peak}} = (197 \pm 6)$, with an amplitude $A_{l_{\text{peak}}} = (69 \pm 8) \mu K$. This is consistent with that expected for cold dark matter in a flat (Euclidean) Universe, as favoured by standard inflationary models.

Photons in the early Universe were tightly coupled to ionized matter through Thomson scattering. This coupling ceased about 300,000 years after the Big Bang, when the Universe cooled sufficiently to form neutral hydrogen. Since then, the primordial photons have travelled freely through the Universe, redshifting to microwave frequencies as the Universe expanded. We observe these photons today as the cosmic microwave background (CMB). An image of the early Universe remains imprinted in the temperature anisotropy of the CMB. Anisotropies on angular scales larger than $\approx 2^\circ$ are dominated by the gravitational redshifts the photons undergo as they leave the density fluctuations present at decoupling. Anisotropies on smaller angular scales are enhanced by oscillations of the photon–baryon fluid before decoupling. These oscillations are driven by the primordial density fluctuations, and their nature depends on the matter content of the Universe.

In a spherical harmonic expansion of the CMB temperature field, the angular power spectrum specifies the contributions to the fluctuations on the sky coming from different multipole $l$'s, each corresponding to the angular scale $\theta = \pi l / a$. Density fluctuations over spatial scales comparable to the acoustic horizon at decoupling produce a peak in the angular power spectrum of the CMB, occurring at multipole $l_{\text{peak}}$. The exact value of $l_{\text{peak}}$ depends on both the linear size of the acoustic horizon and on the angular diameter distance from the observer to decoupling. Both these quantities are sensitive to a number of cosmological parameters (see, for example, ref. 4), but $l_{\text{peak}}$ primarily depends on the total density of the Universe, $\Omega_0$. In models with a density $\Omega_0$ near 1, $l_{\text{peak}} \approx 200 \Omega_0^{1/2}$. A precise measurement of $l_{\text{peak}}$ can efficiently constrain the density and thus the curvature of the Universe.

Observations of CMB anisotropies require extremely sensitive and stable instruments. The DMR instrument on the COBE satellite mapped the sky with an angular resolution of $\approx 7^\circ$, yielding measurements of the angular power spectrum at multipoles $l < 20$. Since then, experiments with finer angular resolution have detected CMB fluctuations on smaller scales and have produced evidence for the presence of a peak in the angular power spectrum at $l_{\text{peak}} \approx 200$.

Here we present high-resolution, high signal-to-noise maps of the CMB over a significant fraction of the sky, and derive the angular power spectrum of the CMB from $l = 50$ to 600. This power spectrum is dominated by a peak at multipole $l_{\text{peak}} = (197 \pm 6)$ ($1\sigma$ error). The existence of this peak strongly supports inflationary models for the early Universe, and is consistent with a flat, Euclidean Universe.

The instrument

The Boomerang (balloon observations of millimetric extragalactic radiation and geodesics) experiment is a microwave telescope that is carried to an altitude of $\approx 38$ km by a balloon. Boomerang combines the high sensitivity and broad frequency coverage pioneered by an earlier generation of balloon-borne experiments with the long ($\approx 10$ days) integration time available in a long-duration balloon flight over Antarctica. The data described here were obtained with a focal plane array of 16 bolometric detectors cooled to 0.3 K. Single-mode feeds provide two $18^\circ$ full-width at half-maximum (FWHM) beams at 90 GHz and two $10^\circ$ FWHM beams at 150 GHz. Four multi-band photometers each provide a 10.5' $14^\circ$ and 13' FWHM beam at 150, 260 and 400 GHz respectively. The average in-flight sensitivity to CMB anisotropies was $140, 210$ and $2,700 \mu K$ at 90, 150, 260 and 400 GHz respectively. The entire optical system is heavily baffled against terrestrial radiation. Large sunshields improve rejection of radiation from $>60^\circ$ in azimuth from the telescope boresight. The rejection has been measured to be greater than 80 dB at all angles occupied by the Sun during the CMB observations. Further details on the instrument can be found in refs 17–21.

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function. The 240 GHz beams of two gaussians. The beams of the total solid angle, due to beam-widths were confirmed as sources. By fitting radial the effective angular resolution and the effects of the 2′ /HM angular resolution of the satellite the CMB power spectrum

240 GHz channels from their pole. The dipole anisotropy has COBE-DMR, fills the beam CMB anisotropies at smaller brator for CMB experiments. peak-to-peak in each 60° scan, and appears in the output of

\[ \text{accuracy of the calibration is ts: uncertainties in the low-}

electronics, and low-frequency, ese is significantly different at \( \pm 10\% \) uncertainty in the angular resolution of the measurement creates an additional uncertainty—indicated by the distance between the points of the red error bars and the blue horizontal lines—that is completely correlated and is largest (11%) at

\[ l = 600. \]

The green points show the power spectrum of a difference map obtained as follows. We divided the data into two parts corresponding to the first and second halves of the timeseries. We made two maps (A and B) from these halves, and the green points show the power spectrum computed from the difference map, \( (A-B)/2 \). Signals originating from the sky should disappear in this map, so this is a test for contamination in the data (see text). The solid curve has parameters \( \Omega_b, \Omega_m, \Omega_A, n_s, h = (0.05, 0.31, 0.75, 0.95, 0.70) \). It is the best-fit model for the Boomerang test flight data, and is shown for comparison only. The model that best fits the new data reported here will be presented elsewhere.

Figure 2 Angular power spectrum measured by Boomerang at 150 GHz. Each point is the power averaged over \( \Delta l = 50 \) and has negligible correlations with the adjacent points. The error bars indicate the uncertainty due to noise and cosmic/sampling variance. The errors are dominated by cosmic/sampling variance at \( l < 350 \); they grow at large \( l \) due to the signal attenuation caused by the combined effects of the 10′ beam and the 14′ pixelization (0.87 at \( l = 200 \) and 0.33 at \( l = 600 \)). The current \( \pm 10\% \) uncertainty in the calibration corresponds to an overall re-scaling of the \( y \)-axis by \( \pm 20\% \), and is not shown. The current \( y \)-uncertainty in the angular resolution of the measurement creates an additional uncertainty—indicated by the distance between the ends of the red error bars and the blue horizontal lines—that is completely correlated and is largest (11%) at

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Fig. 4. — A comparison of the MAXIMA power spectrum with that of the recently reported BOOMERANG experiment (de Bernardis et al. 2000). Consistency between the power spectra has been achieved by scaling the MAXIMA-1 power spectrum down by a factor equal to its 1σ calibration uncertainty and the BOOMERANG power spectrum up by a factor equal to its 1σ calibration uncertainty (the calibration uncertainties are 4% and 10% in ΔT for MAXIMA-1 and BOOMERANG, respectively). These data show a suggestion of a peak at $\ell \sim 525$. Jaffe et al. (in preparation) give a detailed analysis of the combined MAXIMA and BOOMERANG data sets.
But even the spectacular accuracy of MAP and PLANCK give an iso-l(1) line on the Omega(M) - Omega (Lambda) plot.

Fortunately this ambiguity can be resolved by a totally independent set of observations.
III HIGH-Z SUPERNOVAE IA

In recent years several supernovae (Type IA) have been discovered with red-shift \( Z > 0.3 \) (more than 50 of them).

An example of one high red-shift is \( Z = 0.83 \). How far is that in cosmic look-back time? For matter-domination:

\[
\left( \frac{R_0}{R_t} \right) = \left( \frac{t_0}{t} \right)^{2/3}
\]

Given that \( R_0/R_t = (1 + Z) = 1.83 \), then \( t_0 = 2.47t \) and hence \( t \approx 6Gy \) for \( t_0 = 14Gy \).

This is older than the Solar System and more than half-way back to the Big Bang.
These SN are discovered by a 4m. Telescope with CCD.

- Then the light curve is followed for a month or two by the 10m. KEK telescope on Mauna Kea and/or the HST.
- The light curve is key.
- The width of the light curve is an excellent indicator of intrinsic luminosity (by study of low-Z SN).
Broader (slower) light curve implies brighter at maximum

- Clever techniques compare SN light curve to standard light-curve template
- These SN are very far away (over 50% back to Big Bang) but not comparable to the CBR which is 99.998% back to the Big Bang.
Because of the high $Z$ even one observation can influence the measure of the \textbf{DECELERATION PARAMETER} $q_0$:

$$q_0 = -\frac{\ddot{R}}{R^2} = -\frac{\ddot{R}}{R H_0^2} \quad (at \quad t = t_0)$$

In a Hubble plot $Z = d/H_0$ where $d = \text{luminosity distance}$.
If $\Omega_M$ and $\Omega_\Lambda$ are the only energy density sources driving acceleration/deceleration then:

$$q_0 = \frac{1}{2} \Omega_M - \Omega_\Lambda$$

On (II-14) we add a line for $q_0 = -1/2$ suggested by the supernovae observations, to resolve the ambiguity discussed earlier in the context of CBR.

(See next transparency)

Note that a positive $\Omega_\Lambda$ is like a negative pressure tending to accelerate expansion. In thermodynamics decrease of $V$ normally requires work and increases (positive) $P$: here, doing work increases $V$ and hence corresponds to negative $P$. 
\[ \Omega_a = \frac{1}{2} \Omega_m + \frac{1}{2} \quad (q_0 = -\frac{1}{3}) \]

\[ l_1 = 210 \]

Figure 3.

PHF+Ng+Rohm
astro-ph/9806118
MPL A13, 2541 (1998)
PRESENT STATUS

• Clearly more data are needed for both CBR and SNIA.
• The present analysis favors $\Omega(M) \sim 0.3; \Omega(\Lambda) \sim 0.7$.
• $\Lambda$ 120 orders too small?
• Non-zero $\Lambda$ implies we live in a special era: $\Lambda$ negligible in past, dominant in future.
MAIN POINT OF OUR WORK (with Ng and Rohm)

- The value of l(1) depends almost entirely on the geodesic geometry since last scatter and not sensitively on details of acoustic waves.

P.F., Y. Jack Ng, Ryan Rohm
MPL A13, 2541 (1998)
astro-ph/9806118
esp. FNR (29)