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CANCELLATION OF SUDAKOV LOGARITHMS IN RADIATIVE DECAYS OF QUARKONIA

work in progress
in collaboration with S. Catani

Contents of this talk:

- consider photon spectrum in quarkonia decays:
  \[
  \frac{d\Gamma}{dz} (H \rightarrow \gamma + X) \quad (z \equiv 2 E_{\gamma}/M)
  \]

- study endpoint region $z \rightarrow 1$; sensitive to effects of infrared QCD radiation

- \( \Rightarrow \) present results of an analytic, resummed calculation of these effects
PERTURBATIVE QCD APPROACH TO RADIATIVE QUARKONIA DECAYS:

\[
\frac{1}{mv^2} \gg \frac{1}{m}
\]

(scale over which \(q\) and \(\bar{q}\) bind into the quarkonia) \(\quad\) (scale over which the \(q\bar{q}\) pair decays)

Then: expand about \(v \to 0\)

\[
\frac{d\Gamma}{dz} \sim (\text{nonperturbative factor (describing bound state dynamics)}) \times (\text{short-distance factor (describing annihilation)}) + O(v^2)
\]

\[
\sum_n e_n(z) a^n
\]

NLO calculation available: Krämer (1999)

[for "direct" term, color-singlet]

BUT:

THIS SCHEME IS BOUND TO BREAK DOWN AS \(z \to 1\)!
NEAR THE EXCLUSIVE BOUNDARY $z = 1$:

- RELATIVISTIC CORRECTIONS (HIGHER ORDER IN $\alpha_s$) ENHANCED BY POWERS OF $1/\alpha_s$
  
  Color-octet Fock states:
  
  Rothstein-Wise (1994)
  Multipartetelli (1994)

- POTENTIALLY LARGE TERMS IN ln$(1-z)$ IN THE COEFFICIENTS OF THE $\alpha_s$-EXPANSION
  
  T. Phutiadis (1995)
  
  In particular: $\exp(-x^2+\alpha_s x^3)$ from
  
  Phenomenological contribution theory

⇒ RELIABLE RESULTS ONLY AFTER RESUMMATION OF ENHANCED CONTRIBUTIONS

Symptom of sensitivity to infrared scales

Note:

Corresponding effects seen in hadronization models:

R. Field (indep.-fragmentation MC model)

Consoli-Field (effective gluon mass)

PHYSICAL ORIGIN OF ENDPOINT BEHAVIORS: SOFT COLOR RADIATION

This is the subject of this study.

⇒ Main practical outcome: ln$(1-z)$ cancel order by order in $\alpha_s$

for the color-singlet Fock state

(⇒ see next)
STRUCTURE OF SOFT EMISSION

Born: \( \mathcal{H}(P) \rightarrow \gamma(K) + g(K_1) + g(K_2) \)

Coherence of color radiation: [see, e.g., Dokshitzer et al., Basics of perturbation, 2019, 1992]

Feynman graphs (with interferences, loops, etc.) \( \rightarrow \) Branching graphs (with angular ordering)

Jet mass distribution:

\[
J_g((K_1 + K_2), K_1^2) = \prod \frac{d^4 q_i}{(2\pi)^3} \delta(q_i^2).
\]

DL approximation:

\[
\int K^2 \frac{d^2 \mathcal{J}}{T^2} d^2 x e^{-d^2 x^2} \delta^2 \mathcal{K}^2
\]

Soft factorization formula:

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dz} = \int d^4 k_1 d^4 k_2 \delta((P - k_1 - k_2) \cdot z - \frac{2P \cdot k}{M^2}) \delta_+(K_2^2)
\]

\[
\times M_0(P, K_1, K_2) \mathcal{J}_g((K_1 + K_2)^2, K_1^2) \mathcal{J}_g((K_1 + K_2)^2, K_2^2)
\]

Valid to all perturbative orders with LL and NLL accuracy.
CANCELLATION MECHANISM

APPROXIMATE THE PHASE SPACE IN THE COHERENT REGION

\[ k_1^2, k_2^2 \ll (k_1 + k_2)^2 \ll M^2 \]

\[ \Rightarrow M^2(1-2) \]

Boundary on \( x_1 \) comes from:

1) fragmentation of jet 1 \( \rightarrow x_1 \geq \frac{4 k_1^2}{M^2} \)

2) recoiling jet 2 \( \rightarrow x_2 \geq \frac{k_2^2}{M^2(1-2)} \)

THEN

\[
\frac{1}{F} \frac{dF}{dz} = \int_0^1 dx_1 \int_0^{\min\{M^2x_1^2/4, M^2x_1(1-2)\}} dx_2 \ M_0(z, x_1, x_2) \delta(z-2+x_1+x_2) \\
\int dx_2 \ T_g(M^2(1-2), k_1^2) \int dx_2 \ T_g(M^2(1-2), k_2^2)
\]

Plug in the DL expression as an illustration and get

\[
\int_{1-2}^{1} dx_1 \ \ln \left( \frac{M^2(1-2)}{M^2x_1(1-2)} \right) = -\int_{1-2}^{1} dx_1 \ \ln \ x_1
\]

No logs of 1-2 in \( \frac{dF}{dz} \)!

(But there are logs in \( \frac{d^2F}{dz \ dz} \to \))
COMPUTING SUDAKOV LOGARITHMS IN THE DERIVATIVE OF THE SPECTRUM

1-loop calculation:

\[
\frac{1}{\Gamma} \frac{d}{dz} \frac{d\Gamma}{dz} (z) = \left[ -4 \ln (1-z) - 10 + O(1-z) \right] + \\
+ \frac{\alpha_s}{\pi} \left[ \frac{2}{3} C_A \ln^3 (1-z) + \left( \frac{23}{6} C_A - \frac{2}{3} \frac{T_R N_f}{\pi} \right) \ln^2 (1-z) \right] + O(\alpha_s^2)
\]

Resummed calculation:

**SOFT FACTORIZATION FORMULA FOR** \( \frac{d}{dz} \frac{d\Gamma}{dz} \):

\[
\frac{d^2 \Gamma}{dz^2} = \int_0^1 dx_1 \int_0^1 dx_2 \left[ \delta(2-2+x_1+x_2) \Theta(x_1+x_2-1) \right] \cdot \\
\cdot \left[ M_0' + M_0 \frac{\alpha_s}{2\pi} C_A \left( \frac{1}{x_1} \ln x_1 + \frac{1}{x_2} \ln x_2 \right) \right] + 2 \epsilon
\]

**RESUMMATION CURES THE LOGARITHMIC DIVERGENCE** THAT APPEARS IN THE FIRST DERIVATIVE OF THE SPECTRUM AT ANY FIXED ORDER OF PERTURBATION THEORY
DISCUSSION AND CONCLUSIONS

* In the boundary kinematics:

ANALOGY WITH \( e^+ e^- \): "TWO-JET" REGION

→ BUT THE RECOILING SYSTEMS ARE COLOR NEUTRAL!

\[ \downarrow \]

no Sudakov logs → constant shift from \( p_T \)

* CANCELLATION DOES NOT OCCUR FOR COLOR-OCTET:

⇒ IMPORTANT CONSEQUENCE OF 

THE RESULT OF THIS PAPER:

\[ \text{octet} \quad \text{is smaller than expected from power counting at fixed order} \]

⇒ Rothstein-Wise
Maltoni-Petrelli
Gremm-Kapustin

due to different large-order behavior in the two channels

* COHERENCE SCALE IS NOT \( M^2 \) BUT \( M^2(1-\varepsilon) \) ⇒

⇒ destructive interference outside cone \( \Theta^2 \)

(not implemented in currently used MC's ⇒ \( \Theta^2 \))