Sufficient Extra Dimensions

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This work is dedicated to the memory of Miss Barbara Lynn Oldack
Outline

• Beyond Dirac
  – SU(3) as a 3D matrix algebra
  – SM 1-brane states and the oscillating neutrino
    physics/9712042
• Extra Dimensions
  – R-S scaling and the cosmological ‘constant’
    astr-ph/0007100
• AdS/CFT correspondence
  – Higgs boson and a black hole ‘singularity’
M (for Matrix) Theory

Has established the ‘superset of all possible theories’ by
- proving there is a connection between pre-existing theories
- must include the standard model (ref E. Witten “Reflections on the Fate of Spacetime” Phys Today Apr 96, “Duality, Spacetime and Quantum Mechanics” May 97, etc)

Includes D#-brane objects
- some supersufficient to SM (>11 dimensions total)
- how to break symmetries to achieve SM is poorly understood
- compactification onto Calabi-Yau manifolds (ref Green, Morrison & Strominger “Black Hole Condensation and the Unification of String Vacua” hep-th/9504145)

Changes from ‘closed oriented heterotic string’
- orientation vector - better options which may allow separation of quanta
- has a surface area (ref DJ Gross “Nonperturbative Two-Dimensional Quantum Gravity” PRL 8 Jan 90)
Beyond Dirac

- M-theory must extend the Schrödinger / Heisenberg / Dirac pictures to an explicit 11-dimensional picture

- Consider generalized Pauli matrices, $\sigma_i$, and anti-commuting matrices $\rho_i$, neglecting time (to understand the combinatorial algebra of 3+6 dimensional D1-brane states)

- Let $\alpha_1 = \rho_1 \sigma_1$ represents the 3X3 matrix operation:

$$\begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}$$


- Define $R \equiv \alpha_1^T \equiv (\rho_1 \sigma_1)^T$, a 90° rotation operator
3X3X3 matrix notation

To get $SU(3)_c \times SU(2)_L \times U(1)_y \rightarrow$ use restrictions

\[ \begin{pmatrix} abc \\ def \\ ghi \end{pmatrix} \begin{pmatrix} jk l \\ mno \\ pqr \end{pmatrix} \begin{pmatrix} st u \\ vwx \\ yzA \end{pmatrix} \rightarrow \begin{pmatrix} gda \\ heb \\ ifc \end{pmatrix} \begin{pmatrix} jk l \\ mno \\ pqr \end{pmatrix} \begin{pmatrix} st u \\ vwx \\ yzA \end{pmatrix} \]

Using this 3X3X3 matrix notation is both very cumbersome AND - it doesn’t preserve rotations of each element - “a” thru “A” (these rotations will be used in lieu of complex numbers to represent anti-particles)

But a better notation does exist, which captures the full beauty of $SU(3)_c \times SU(2)_L \times U(1)_y$.

The matrix operator $R$ is applied above-

and only corner-position-preserving operators are allowed.

i.e. operators similar to $\langle I | (RG \cdot R \cdot G) | I \rangle$ (Def: R, G, B faces $\rightarrow$ )
The 8-fold Way - colorized

The Standard Model showing color- and charge-symmetries SM QCD coloring *assigned* (i.e. broken symmetry).
Rishon combinatorial notation included, with position symmetry kept.
Modeling Quarks, Particles & Strong Interactions

The path integral approach in the generalized notation of string theory:
(ref Quantum Fields and Strings Vol 1, E. Witten et al, p505)

\[
\int_{X:T^2 \rightarrow M} DXD\Psi_+ D\Psi_- e^{-\mathcal{L}}
\]

An equivalent formula in Dirac abstract vector notation,
Compactification to a Calabi-Yau threefold

\[
[(G^\prime CGC^\prime)(C^\prime YCY^\prime)(Y^\prime GYG^\prime)]^2 \{ G_{g-b} = B^\prime GM^\prime Y(CB^\prime C^\prime B)^2 Y^\prime MG^\prime B \}
\]

\[
|H> < \alpha | \Omega | \beta >
\]

gives \(ru, gu, bd\) proton
Compactification must involve Orthogonalization

SM Quanta must be both separable and innately described by the string (1-brane)

- **Spin** (Ref Tomonaga “The Story of Spin”, review in Phys Today Feb 98 & Pauli’s Spin Statistics Theorem)
- **Color charge** (usual QCD conventions)
- **Electric charge**
- **particle or anti-particle** (CP, CPT violation?)

..mass must be unified in the theory in a way which explains the existence of several mass-coupling constants (and eventually calculate their respective values a priori)
A string (1-brane) with standing “gravitational waves”

Standing waves have non-perturbative maxima which are equated to color quantum, based on spin orientation - can change; strong & weak interaction
- compactification onto C-Y 3-folds

Electric charge is identified by the curvature \((\pm \varnothing)\) (convex, concave relative to the particle’s spin/motion)

Spin quantum is defined only for the composite particle

Chirality wrt to orientation vector defines particle vs. antiparticle
Spin -1/2     Spin 1/2

cd = ru

quark

Spin is determined by the orientation of intrinsic spin to chiral “orientation” vector

3x3x3 matrix algebra has 1-1 correspondence to ‘orientifolds’:

\[ T \]

charge 1/3

charge 2/3

ru quark

T carries charge 1/3

Spin 1/2
Recall that these are all *now* massive particles.

The orientation (color chirality) vector indicates that $dA$ has physical meaning... i.e. mass

$$m = G_{\mu\nu} e^{-\lambda} |(G^{-CGC^{-}})(C^{-YCY^{-}})(Y^{-GYG^{-}})|^2 >$$

R-S scale factor $\lambda = \gamma \alpha(r_R, r_G, r_B, \phi_R, \phi_G, \phi_B)$

$$\sim k \frac{R_5}{R_3}$$

Beth, mass per 1-brane unit area, $\alpha'$, should have a formulation derived from a 2-D Quantum Gravity approach

(ref DJ Gross, PRL Vol 64 #2, p127 Jan 90.)
Weak Interactions

\[ \int \Psi^*_\alpha(r) \Psi^*_\beta(r) d^3r = \sum_\Gamma \langle \alpha | r \rangle \langle r | \beta \rangle \]

(Ref Shiff, Eqn 23.26’)

Where the intermediate vector bosons \( W^\pm, Z^0 \) are

[CR\(^-\)MG\(^-\)R\(^2\)](YR\(^-\)Y\(^-\)R\(^2\))(RM\(^-\)R\(^-\)M\(^2\))[R\(^2\)GM\(^-\)RC\(^-\)]

\[ \equiv \langle \alpha | r \times r | \beta \rangle \]

One of many… the \( \Sigma \) is the usual Feynman

SM formulae are recovered intact!

Here the vertex operator and propagator are specific, representing an explicit interaction, here on \( ru + \bar{rd} \)

may help resolve Pauli spin statistics problem …?

**NOTE:** the underlying algebra unifies Strong & Weak!
Closed vs Open strings

- most of mass comes from the gluon
- in both cases proportional to length
- but how to recover QCD combinatorics?
Oscillating, massive neutrinos

The trefoil knot is fundamental to string theory

$\nu_s \leftrightarrow \nu_\tau$

can’t break the string

Chargeless & colorless (VVV)-area is inherently extremely small

Neutrino - can interact-large mixing angle!

$\langle B^2 | (M^* Y M Y^*)^2 (Y^* C Y C^*)^2 | B^2 \rangle$

electron, neutrino+gluon

The Tripartite String Photon

no area $\therefore$ no mass
Complex Gravity

\[ e^{i\theta_{\mu\nu} - \lambda} \left| D_{3x3x3} \right\rangle \]

\[ \sum_U (G_{\mu\nu} + iB_{\mu\nu}) \text{e}^{-\lambda \left[ \left( G^\dagger CGC \right) \left( C^\dagger YCY \right) \left( Y^\dagger GYG \right) \right]^2} = 1 \]

\[ \lambda = \gamma \alpha(r_R, r_G, r_B, \Phi_R, \Phi_G, \Phi_B) \]

At a fixed time, summing over all particles in space, this reduces to a formula which is equivalent to

\[ \Omega_m + \Omega_\Lambda \]

So the model now has 9-D space plus 2-D “imaginary time”
(necessary to write boundary conditions for cosmology)

This result inspired by Strings 2000 conference presentation
A flat universe: \( \Omega_m + \Omega_\Lambda = 1 \)

Consider a cyclic cosmology model

\( \Omega_m \) is area-like for consistency with string theory, and such that mass is orthogonal to other intrinsic string properties.

\( \Omega_m = \sum_U f(\alpha) \),

mass is a function of 1-brane cross-section.

\( \Omega_\Lambda \) is not a constant, but slowly varying.
\[ T_n = \frac{\ln(T/T_o)}{42} \]

\[ T_{\text{end}} \approx 200 \text{ trillion years} \]

\[ T_o = 10^{-9}s \]

C) Current ‘epoch of mini-inflation’ explained by ‘residual white holes’

A) Guth’s inflationary cosmology

B) Hawking’s Black Holes

Planck scale

Both axes use R-S scaling

\[ (M_{\text{univ}} - M_{\text{bh}})/M_{p,\text{max}} \]

\[ (M_{\text{univ}} - M_{\text{wh}})/M_{p,\text{max}} \]
The Inflationary Big Bang model

- General Relativity applies at all times
- symmetries broken spontaneously at $10^{27}$ °K
- requires a critical mass to reach closure

Recent results:

- mirror symmetry modifies Relativity at event horizon
  $\rightarrow$ cosmological constant; $F_g = G/(r^2 - \alpha')$  ..Maldacena
  (Greene, Morrison, Strominger)
- “drastic change in the structure of matter at $1.5 \times 10^{12}$ °K”
  (Wilczek cites Karsch)
- “The Universe Will Expand Forever” - no missing mass
  (Permutter & Garnovich)
Cyclic Cosmological model

- No critical mass is required for closure
- time when ‘final’ chiral symmetry is broken, corresponding to $T=1.5\times10^{12}\,\text{oK}$, is $t_0=10^{-6}\text{s}$
- confinement broken earlier ($10^{-15}\text{s}$)
- smooth transition from inflation to standard expansion ..mini-inflation

“A repulsive force in the universe seems to be at work on a cosmic scale.” per astronomical observations by LBL (see Jan AAS)

- would agree with cyclic model, $t_0 = 10^{-9}\text{s}$

- Mirror symmetry (GM&S) means that “superstrings, black holes, and elementary particles.. might be the same”
Higgs Boson

Planck or $\sqrt{\alpha'}$

Requires that the string or 1-brane have 3 ‘partitions’, in agreement with M. Kaku at interaction vertices - but exists only for $\sim 10^{-42}$s !!

Compactification to a Calabi-Yau threefold-
Kachru, Lawrence, Silverstein
but without true singularities in the theory

Supersymmetry (between bosons and fermions) is intact
The kernel of a Black Hole as related to a Higgs boson ->
Black Hole ‘singularity’ in the Tripartite String model

$S_{BH} = \frac{1}{4} \text{Area}$

Planck $= \sqrt{\alpha'}$, or $10^{-19} \text{m}$?

Bekenstein-Hawking area-entropy law appears to be in agreement with Hayward, Mukohyama & Ashworth ‘entanglement entropy’ when strings (1-branes) are densely entangled on the surface of the black hole

NOTE-> BILATERAL ASYMMETRY
No known astrophysical objects are bilaterally asymmetric
At ‘singularity’ particles “freeze in time” - appear to be ejected