A COMMENT ON THE BLACK HOLE INFORMATION PARADOX

SAMIR D. MATHUR
OHIO STATE UNIVERSITY

WHEN CAN WE IGNORE EFFECTS OF QUANTUM GRAVITY?

String theory: black hole formation and evaporation is a unitary process
Semiclassical Hawking calculation: loss of information

→ AN EXTRA LIMITATION ON THE DOMAIN WHERE WE CAN IGNORE QUANTUM GRAVITY

(s.a.m.: gr-qc 0807111 [IJMP, to appear])
2. Domain where we believe we can ignore quantum gravity

(a) Space is gently curved
\[ |R| \ll \frac{1}{\ell_{\text{planck}}} \]

(b) Spacelike slicing is smooth

(b1) Intrinsically curvature is small
\[ |(\mathcal{S})R| \ll \frac{1}{\ell_{\text{planck}}} \]

(b2) Extrinsic curvature is small
\[ |K| \ll \frac{1}{\ell_{\text{planck}}} \]

(c) Matter is long wavelength
\[ \lambda > > \ell_{\text{planck}} \]

Then \[ i\hbar \frac{d}{dt} \bar{\psi} = H \bar{\psi} \]

Physics local, "standard"
MANY FINGERED TIME IN GENERAL RELATIVITY

FLAT SPACE

??
Can we duplicate an object in a quantum mechanical evolution?

→ No "quantum xeroxing"

\[ |\Psi_1\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi_1\rangle \]
\[ |\Psi_2\rangle \rightarrow |\Psi_2\rangle \otimes |\Psi_2\rangle \]

\[ |\Psi_1\rangle + |\Psi_2\rangle \rightarrow |\Psi_1\rangle \otimes |\Psi_1\rangle + |\Psi_2\rangle \otimes |\Psi_2\rangle \]

Or \((|\Psi_1\rangle + |\Psi_2\rangle) \otimes (|\Psi_1\rangle + |\Psi_2\rangle)\)

But we escape the problem:

Slice becomes timeline
But in a black hole metric, can make a foliation such that:

(1) All slices are spacelike, satisfy smoothness conditions (a) – (c)

(2) On initial slice, have matter that made black hole

On later slice, have matter that made black hole as well as Hawking radiation that should have information of this matter

Need an extra criterion for validity of semiclassical physics (no quantum gravity)

→ Slices of foliation should not stretch too much
CONSTRUCTING THE FOLIATION

\[ ds^2 = -(1-2\frac{M}{r})dt^2 + \frac{1}{r} dr^2 + r^2 d\Omega^2 \]

SPACELIKE SLICE:

\[ t = \text{constant}, \quad 4M < r < \infty \]
\[ r = \text{constant} = M \quad (\text{inside horizon}) \]

Smooth "connector" region
"NEGATIVE ENERY" PARTICLES

HAWKING RADIATION

INITIAL MATTER

\( r = 0 \)

\( r = 2m \)

\( r \to \infty \)
For unitarity, need nonlocal information transport.

- How does nonlocal information transport happen?

- When does nonlocal information transport happen?

Smoothness conditions (a) - (c) satisfied then what is the "trigger" that tells us that new physics may be invoked.
SLICES IN FOLIATION STRETCH BY A LARGE AMOUNT

A HEURISTIC SCHEME

- SLICE IS DESCRIBED NOT ONLY BY ITS INTRINSIC GEOMETRY AND EMBEDDING, BUT ALSO BY A DENSITY OF DEGREES OF FREEDOM (DoF)

- DoFs are conserved, so stretching slice makes them dilute

- If matter on slice needs more DoFs than available, then nonlocal information transport is possible
CONJECTURE:

In any spacetime (including black hole geometries) take a foliation such that

(a) $R \ll (\ell_p e)^{-2}$

(b) $R \ll (\ell_p l)^{-2}$, $|k| \ll (\ell_p l)^{-1}$

(c) Matter wavelengths $\lambda \gg \ell_p l$

(d) Some matter and its corresponding radiation are captured on the same slice

Then

(e) There will be at least one point where

\[
\text{DOF needed} < 1
\]

\[
\text{DOF available}
\]
COMMENTS:

- On initial slice, estimate count of D.O.F. by Bekenstein formula
  \[ S \sim \frac{\text{area of boundary}}{G_N} \]

- D.O.F. needed \( \Rightarrow \) D.O.F. needed to describe "negative energy" quanta in black hole background.

- Number of particle species involved in Hawking radiation cancels out.

- Checked for a variety of black hole backgrounds in different dimensions.
**Implications**

- Picture consistent with AdS/CFT, since gauge theory has finite d.o.f. But need to show that in string theory we cannot stretch a slice too much without trouble.

**Cosmology:** Space expands by a large factor.

- Dilute d.o.f.; nonlocal info transport? Horizon problem?

\[ S \sim \text{d.o.f.} \sim \frac{\text{Area}}{G_N} \]

Expand space, dilute d.o.f.;

\[ \Rightarrow \ G_N \text{ goes up?} \]
CONCLUSION:

Imlications of black hole information paradox are far reaching, and have not been assimilated in any clear way into our current understanding of string theory.