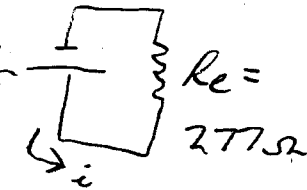
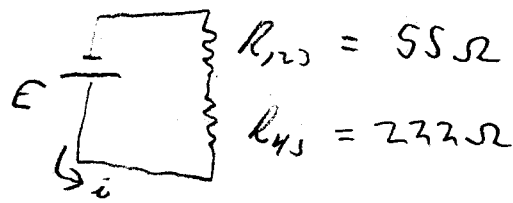
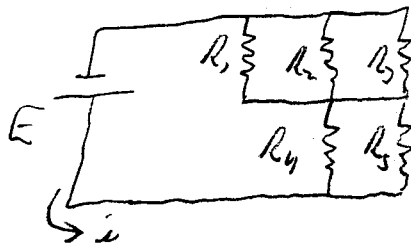


#1)



$$V_1 = V_2 = V_3 = V_{123} = i R_{123} \quad \text{but } i = \frac{E}{R_e} = 0.36 \text{ A}$$

$$= (0.36 \text{ A})(55 \Omega) = 20 \text{ V} \checkmark$$

$$V_4 = V_5 = V_{45} = E - V_{123} = 80 \text{ V} \checkmark$$

$$i_1 = \frac{V_1}{R_1} = \frac{20 \text{ V}}{100 \Omega} = \frac{1}{5} \text{ A} \checkmark$$

$$\text{Likewise } i_2 = \frac{1}{10} \text{ A} \quad i_3 = \frac{1}{15} \text{ A} \checkmark$$

$$\left(\text{Notice } i_1 + i_2 + i_3 = 0.36 \text{ A} = i \checkmark \right)$$

$$i_4 = \frac{V_4}{R_4} = \frac{80 \text{ V}}{400 \Omega} = \frac{1}{5} \text{ A} \quad i_5 = 0.16 \text{ A} \checkmark$$

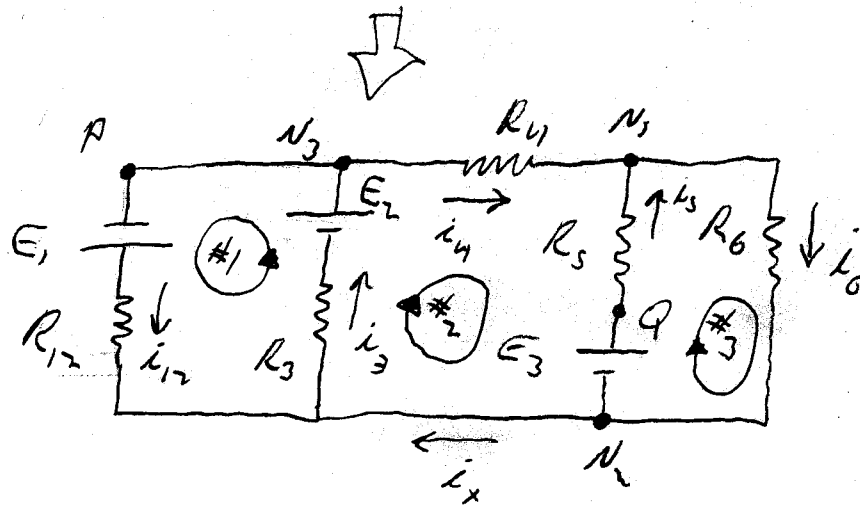
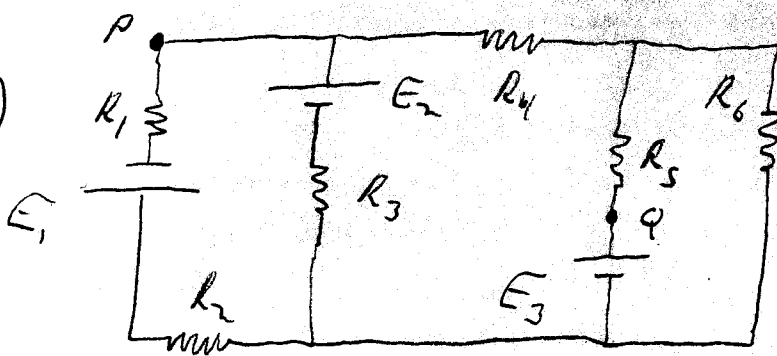
$$\left(\text{Notice } i_4 + i_5 = 0.36 \text{ A} = i \checkmark \right)$$

$$P_{\text{dissipated}} = i^2 R_e = (0.36 \text{ A})^2 (277 \Omega) = 36 \text{ W} \checkmark$$

$$P_{\text{supplied}} = iE = (0.36 \text{ A})(100 \text{ V}) = 36 \text{ W} \checkmark$$

$$\left(\text{Notice: total power used} = \text{total power supplied} \right) \checkmark$$

#29)



$R_{12} = R_1 + R_2$
 I've identified and labeled all currents and picked 3 loops and 3 nodes.

There are 6 unknown currents: i_{12} , i_3 , i_4 , i_5 , i_6 , i_x so I need 6 equations.

(Do you see that $i_x = i_4$? That's a nice insight but I won't use it so I can illustrate the general procedure.)

Loop #1 : $E_1 - i_{12}R_{12} - i_3R_3 + E_2 = 0$

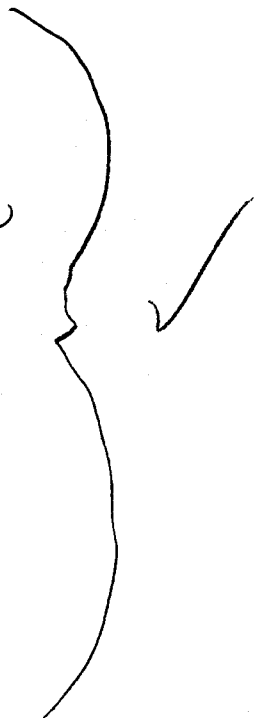
Loop #2 : $-i_3R_3 + E_2 - i_4R_4 + i_5R_5 - E_3 = 0$

Loop #3 : $E_3 - i_5R_5 - i_6R_6 = 0$

Node N1 : $i_4 + i_5 = i_6$

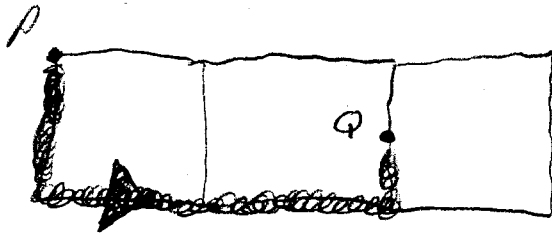
Node N2 : $i_6 = i_5 + i_x$

Node N3 : $i_3 = i_{12} + i_4$



2b)

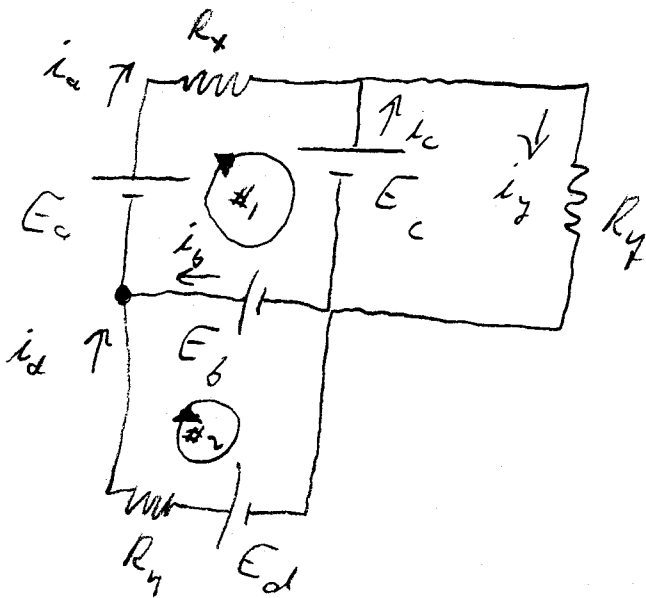
"Walk" from P to Q and keep track of potential differences encountered along the way. Any path will do, but the fewer resistors the better in general. Here's my path:



$$V_P + E_1 - i_{12} R_{12} + E_3 = V_Q$$

$$V_P - V_Q = -E_1 + i_{12} R_{12} - E_3 \quad \checkmark$$

#3) Here's a simpler equivalent circuit:



$$R_x = \frac{1}{3} R_1 = 200 \Omega$$

$$R_y = 100 \Omega$$

Powers We need the battery currents.

$$P_a) P_a = i_a E_a = \left(\frac{1}{10} A\right) (10V) = 1W \checkmark \text{ supplied } \checkmark$$

$$\left. \begin{array}{l} \text{Loop \#1: } E_a - i_x R_x - E_c + E_b = 0 \\ i_x = \frac{E_a - E_c + E_b}{R_x} = \frac{20V}{200\Omega} = \frac{1}{10} A \end{array} \right\}$$

$$P_b) P_b = i_b E_b = (0.225A)(30V) = 6.75W \checkmark \text{ supplied } \checkmark$$

$$\left. \begin{array}{l} i_b + i_d = i_x \text{ (Node rule)} \\ E_d - i_d R_y - E_b = 0 \text{ (Loop \#2)} \\ i_d = \frac{E_d - E_b}{R_y} = \frac{-25V}{200\Omega} = -\frac{1}{8} A \\ i_b + \left(-\frac{1}{8} A\right) = \frac{1}{10} A \\ i_b = 0.225A \end{array} \right\}$$

$$P_d) P_d = i_d E_d = \left(\frac{1}{8} A\right) (5V) = \frac{5}{8} W \checkmark \text{ used } \checkmark$$

$$\begin{aligned}
 P_c) \quad P_c &= i_c E_c \\
 &= \left(\frac{1}{10} A\right) (20V) \\
 &= 2W \checkmark \\
 &\text{supplied } \checkmark
 \end{aligned}$$

$$i_c + i_a = i_y \quad (\text{Node rule})$$

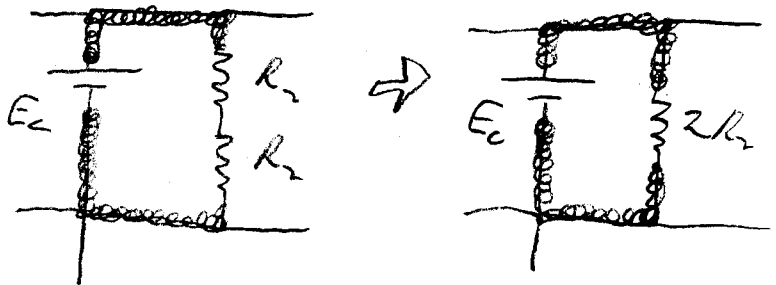
$$i_y = \frac{E_c}{R_y} = \frac{1}{5} A \quad (\text{by inspection})$$

$$i_c + \left(\frac{1}{10} A\right) = \frac{1}{5} A$$

$$i_c = \frac{1}{10} A$$

$$i_{R_1} = \frac{1}{3} i_a = \frac{1}{3} \left(\frac{1}{10} A\right) = \frac{1}{30} A \checkmark$$

$$\begin{aligned}
 i_{R_2} &= \frac{E_c}{2R_2} \\
 &= \frac{20V}{200\Omega} \\
 &= \frac{1}{10} A \checkmark
 \end{aligned}$$

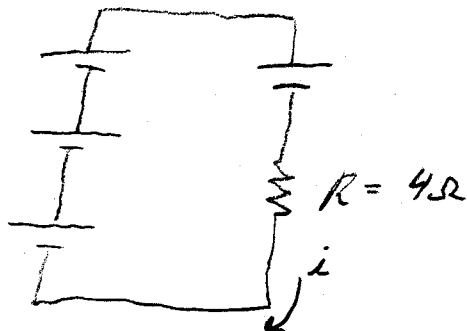


I'm only showing part of the circuit but battery E_c is directly connected to the R_2 resistors.

That's all we need to know.

#4) Look at the path outlined in a thick black line on the figure on the next page.

If we pull that part of the circuit out:



We see the voltage across R is $4V + 4V + 4V - 4V = 8V$.

$$i = \frac{8V}{4\Omega} = 2A \checkmark \quad (\text{Use the loop rule if you don't see this.})$$

Note we cannot tell from this what the currents through the batteries are. You need to consider the entire circuit for that.

#5) Same thing. From the figure, you can see that the circled battery is directly connected to C so

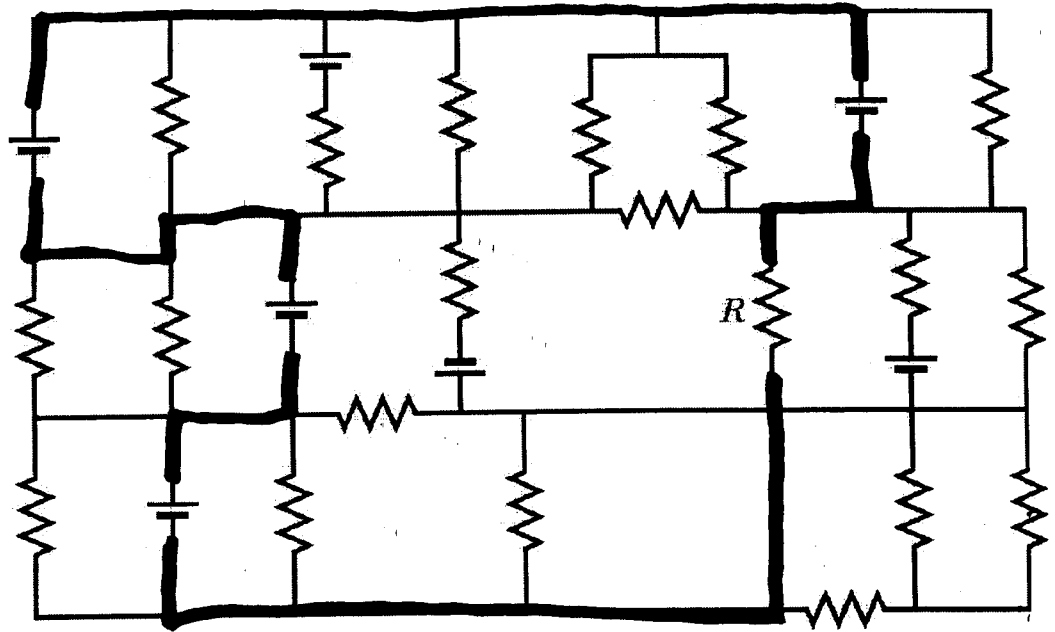
$$q = CV = (10\mu F)(5V) = 50\mu C \checkmark$$

Week #7

Multiloop circuits #2.

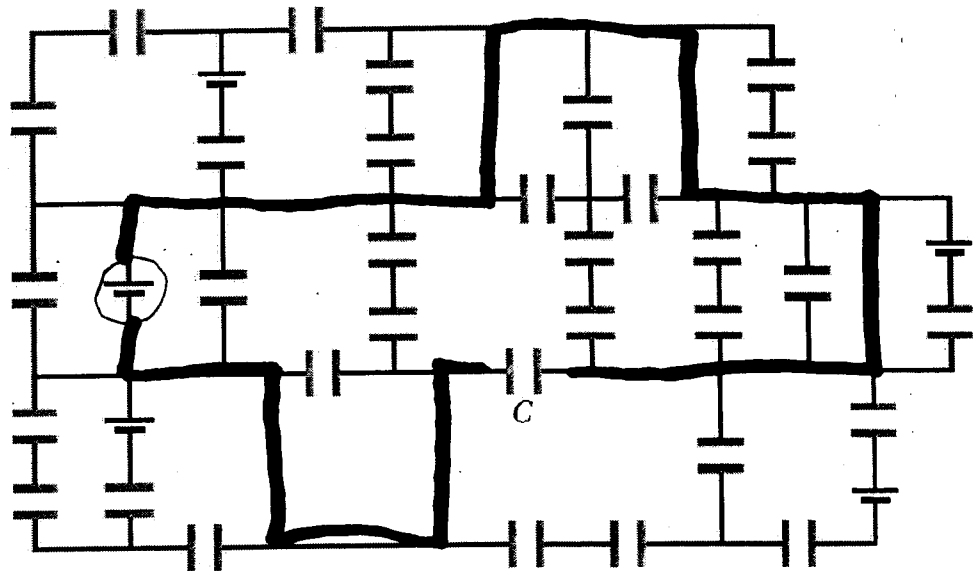
Problem #4. Each resistor is $4\ \Omega$ and each battery is $4\ \text{V}$. Find the current through resistor R .

There's a trick to this. You need to find the potential difference across R . You can do this using the method of "walking" from one end of the resistor to the other and keeping track of the potential differences along the way. The potential difference of a battery is easy. The potential difference of a resistor is harder because you need to know the current through it, so best to avoid resistors during your walk.



Problem #5 Each capacitor is $10\ \mu\text{F}$ and each battery is $5\ \text{V}$. Find the charge on capacitor C .

The text calls problems #4 and #5 circuit "mazes". See why?



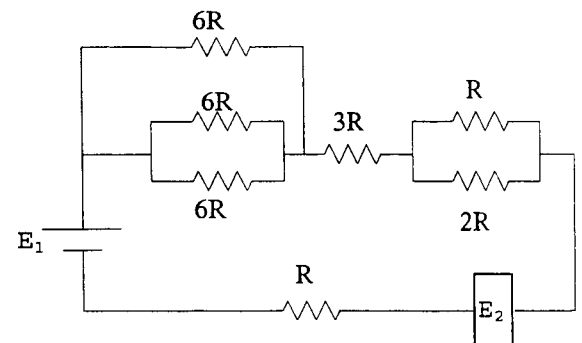
Problem #6

$E_1 = 10\ \text{V}$ and battery E_1 is supplying a power of $10\ \text{W}$.

$R = 100\ \Omega$.

The square on the bottom right-hand-side of the circuit labeled E_2 is a battery.

- What is E_2 ? Is battery E_2 supplying power or charging?
- What is the voltage across the $3R$ resistor?
- How much power is dissipated by the resistors in the circuit?



#6)

1) The current through $3R$ is just i .

$$V_{3R} = i(3R)$$

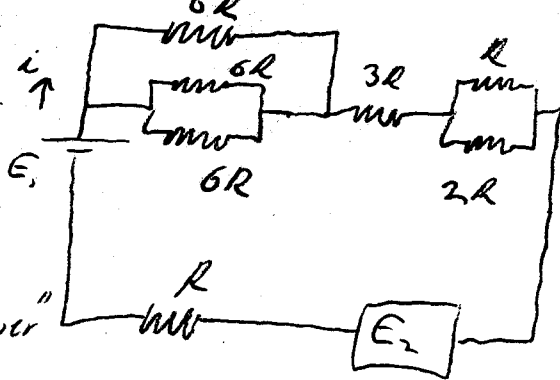
but, $P_{E_1} = iE_1$

$$i = \frac{P_{E_1}}{E_1} = \frac{10W}{10V} = 1A$$

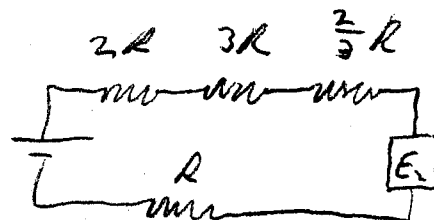
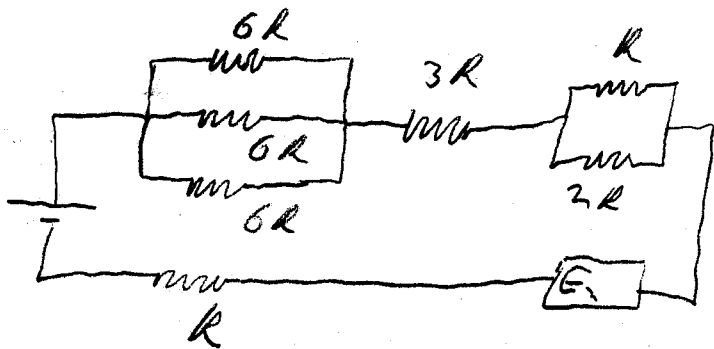
$$V_{3R} = (1A)(300\Omega) = 300V \checkmark$$

Since E_1 is "supplying power" this is the correct current direction.

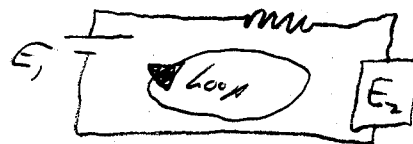
E_2 is going to be a big battery since E_1 only provides 10V!



2) We can relate E_2 to the rest of the circuit using a loop rule. Let's simplify the circuit first.



↓ R_e

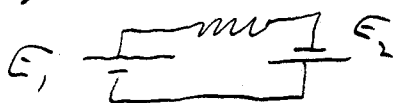


$$R_e = 2R + 3R + \frac{2}{3}R + R = 6\frac{2}{3}R = 667\Omega$$

$$+E_1 - iR_e + E_2 = 0$$

$$E_2 = iR_e - E_1 = 667V - 10V = 657V \checkmark$$

supplying power \checkmark



$$P = iR_e = 667W \checkmark$$

Resistors