Physics 132

Introductory Physics: Electricity and Magnetism

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Recap: Lecture #1 Electric Charge and Coulomb's Law

(Please read so you can start today up to speed. You <u>don't</u> need to write this down. It's a summary of last lecture and is available on the web.)

Electric charge, a "new" quantity, gives rise to the electric force – a "new," fundamental, long-range force.

Rubbing two objects can transfer charge from one to the other.

When two plastic rods were each rubbed with a piece of fur, they repelled. Conclusion: Since they were prepared the same way, they should have the same <u>kind</u> of charge. Thus: "Like charges repel."

On the other hand, the plastic rods were attracted to the fur. There must be a different kind charge such that: "Unlike charges attract."

We call these two kinds of electric charge positive and negative. (We have not observed a third kind of electric charge.)

We measure charge in Coulombs (1 C is a <u>large</u> amount of charge) and denote it with a "q", "Q", "q_1", etc.

The magnitude of the force between two <u>point</u> charges is given by Coulomb's Law:

$$\left|\vec{F}\right| = k\frac{\left|q_1\right|\left|q_2\right|}{r^2}$$ **where k = 8.99 x 10^9 Nm^2/C^2. The direction of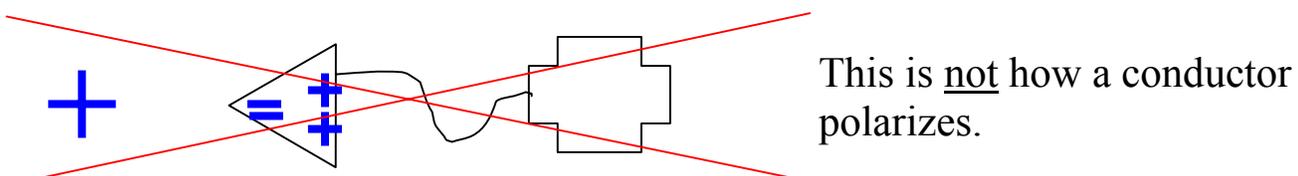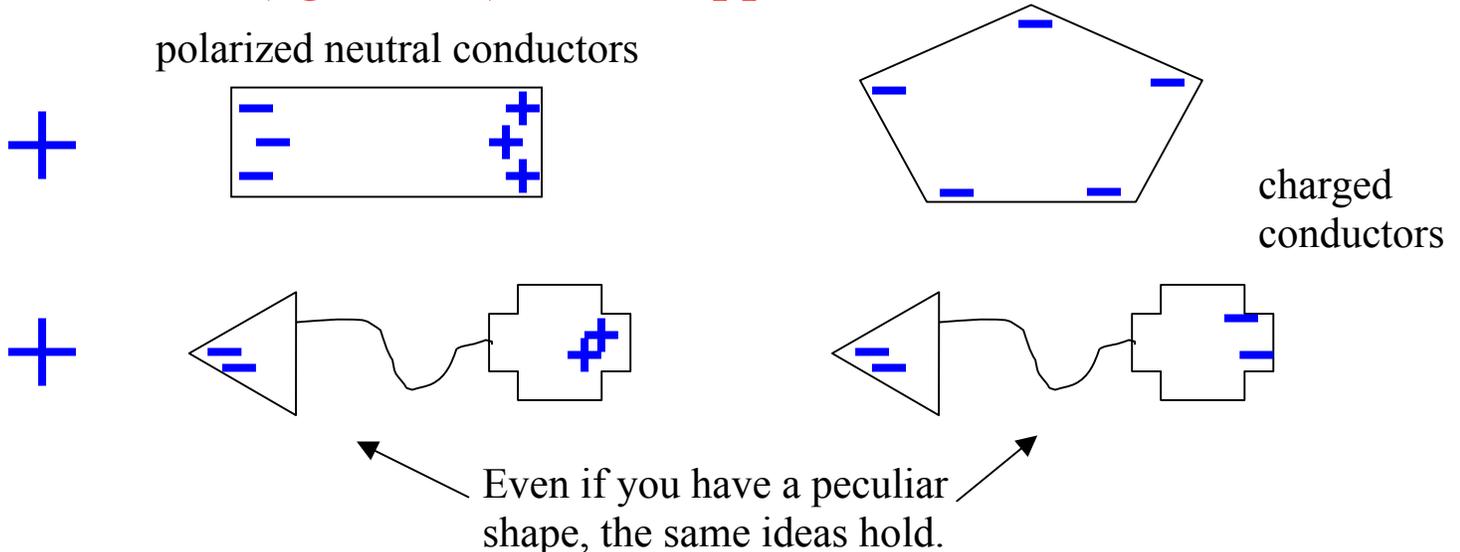 the force is along the line of motion connecting them, either repulsive or attractive. Also:** $\vec{F}_{12} = -\vec{F}_{21}$ **.** F_{12}

force <u>on</u> charge #1 …

… due to charge #2

The electrons that make up an **insulator** can <u>shift a little</u>, so an outside external charge can <u>polarize</u> a neutral insulator. Also, added charge is stuck in place.

polarized neutral insulator

charged insulator

The electrons that make up a **conductor** can move freely. Added charge can also move freely.

- Conductors polarize easily.

- Excess charge spreads out as much as possible because of the repulsion between charges of the same sign.

- Conductors in contact act like a big conductor, no matter how peculiar the shape.

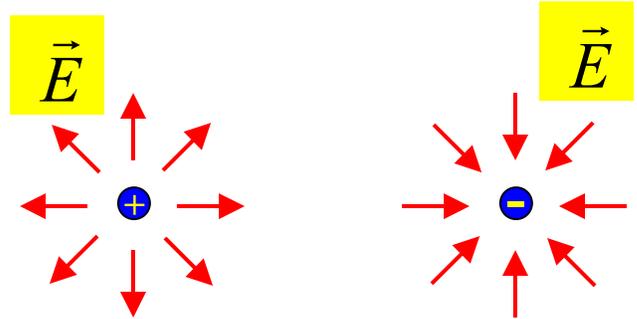- The earth ("ground") can be approximated as a conductor.

polarized neutral conductors

charged conductors

Even if you have a peculiar shape, the same ideas hold.

This is <u>not</u> how a conductor polarizes.

All electric charge generates an electric field. For a <u>point</u> charge q: $E = k \dfrac{q}{r^2}$.

The direction is radial and depends on the sign of the charge.

$\vec{E}$

$\vec{E}$

Electric field has units of N/C.

Since the electric field is a vector, if several charges are present (q$_1$, q$_2$, q$_3$ …), the electric field is just the <u>vector</u> sum of the individual fields from each charge:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

This electric field exerts a force on any other <u>point</u> charge, Q, according to: $\vec{F}_Q = Q\vec{E}$.
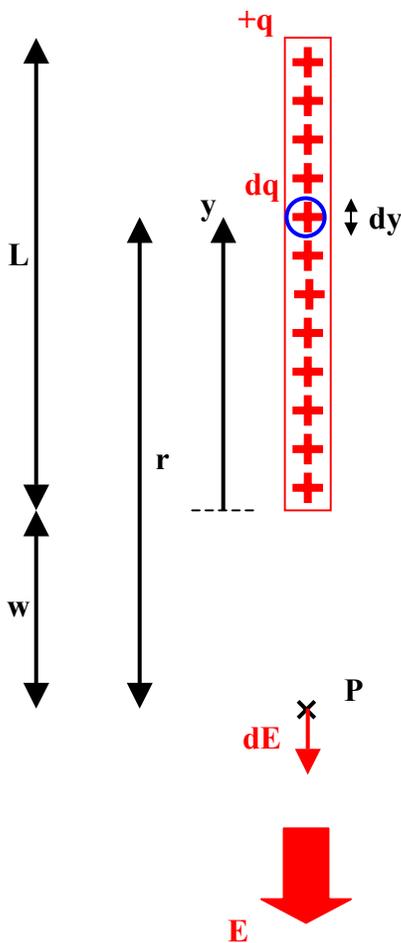
If Q > 0, then the force is parallel to the electric field.

If Q < 0, the force is anti-parallel (opposite) to the field.

Note that the electric field from a <u>point charge</u> follows an inverse square law. If you increase your distance from the charge by a factor of 3, the field falls off by a factor of 9.

Recap: Lecture #4   Continuous charge distributions.

+q

Given: L, w, +q   Find: electric field at P

By symmetry, the electric field acts downward.

Pick a typical "point charge", dq. No matter which point charge is picked, its electric field, dE, also points downward. No need to use components for this problem.

Electric field from the point charge: $dE = k\dfrac{dq}{r^2}$

Electric field from all point charges: $E = \int dE$

Thus, $E = \int k\dfrac{dq}{r^2}$

To make sense of the integral, introduce a coordinate system that allows you to precisely specify where a given point charge is located. Using the coordinate system:

$r = w + y$

$\dfrac{dq}{q} = \dfrac{dy}{L}$ *therefore* $dq = \dfrac{q}{L}dy$

So, we have:
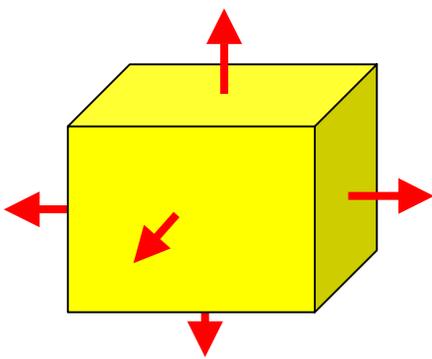
$$E = k\int \frac{1}{(w+y)^2}\frac{q}{L}dy \ = \ \frac{kq}{L}\int_0^L \frac{1}{(w+y)^2}dy$$

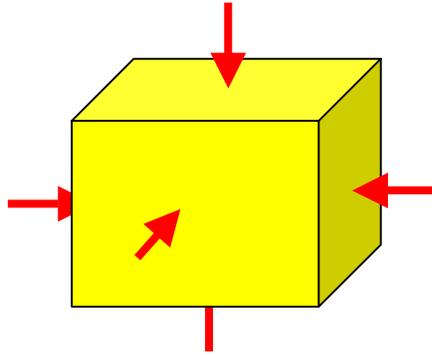**Don't forget to add integration limits!!! They "tell" the integral where the charge is.**
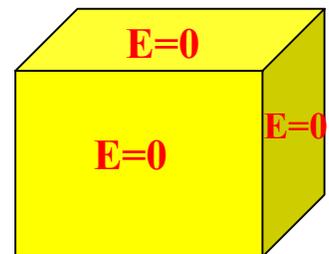
Given a <u>surface</u> (not a solid!) and the electric field <u>on</u> the surface (not inside or outside!), we can determine the charge enclosed by the surface. Here are some examples that show this idea qualitatively:

There must be positive charge inside.   **There must be negative charge inside.**   **There must be zero charge inside.**

Gauss's Law says the same thing quantitatively:

$$\Phi = \frac{1}{\varepsilon_o} q_{enc}$$

Φ = the <u>flux</u> through a closed surface. Flux is essentially the amount of electric field poking through the surface. It can be positive or negative. (A closed surface "holds water".)

q_{enc} = the charge <u>enclosed</u> by the surface. Charge that is outside the surface must not be included.
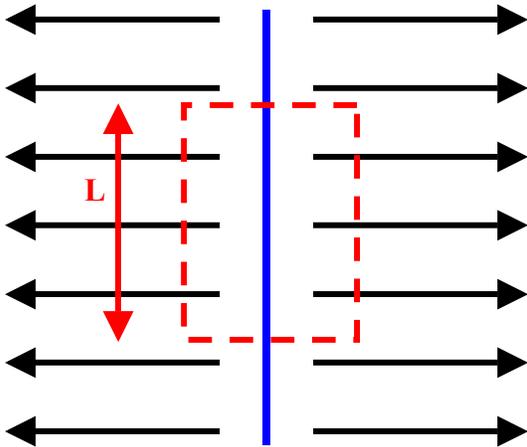
Special-special case: If $\vec{E}$ is remains perpendicular to the surface, no matter how it might curve around, and if it has the same magnitude on the surface: $\Phi = EA$. (A = area.)
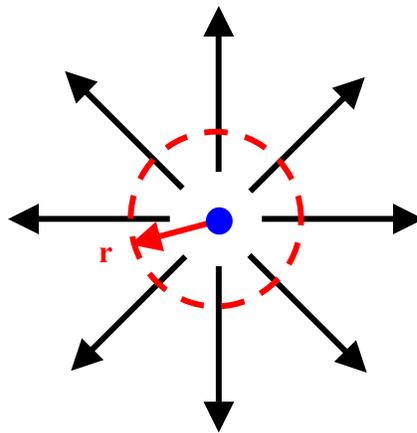
Infinite line charge, linear charge density $+\lambda$.
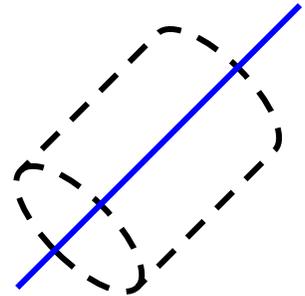
Find the electric field everywhere.

Side view. Infinite line charge in
blue, field lines in black, Gaussian
surface in red.   End view.   Perspective view.

- **Introduce a cylindrical Gaussian surface so that it is everywhere normal to the field lines. Then:**

$$\Phi = EA = E\,(2\pi rL)$$

Note the flux through the end-caps is zero.

- **Use Gauss's Law to find the flux a *second* time:**

$$\Phi = 1/\varepsilon_o \; q_{enc} = 1/\varepsilon_o \; \lambda L$$

- **Combine the results to find the field magnitude:**

$$E\,(2\pi rL) = 1/\varepsilon_o \; \lambda L$$

$$E = \lambda/(2\pi\,\varepsilon_o r)$$

Direction: "radially outwards"

Once a solid charged conductor reaches equilibrium (this happens quickly and is the only case we're considering right now):

The charge resides on the outer surface.
The electric field inside the "meat" of the conductor is zero.

The field inside any <u>empty</u> cavities is also zero. We proved this using Gauss's Law:

Because $E = 0$, the flux through the Gaussian surface is zero: $\Phi = q_{enc}/\varepsilon_o$, therefore the net charge enclosed is zero, therefore the cavity is charge free!

There is a non-zero field on the surface, however.

E≠0   E=0   E=0

Conductor with net positive charge.
Cross-section-view.

If we place a charge inside the cavity, the field inside the meat of the conductor is still zero, but everything else changes:

- **The field inside the cavity is non-zero because of the charges inside it.**

- $q_{enc} = 0$ because $E = 0$ in the "meat", but, $q_{enc} = q_{inner} + q_{cavity}$, so:

  $$q_{inner} = q_{enc} - q_{cavity} = 0 - (-q) = +q!$$

E≠0   E≠0 -q   E=0
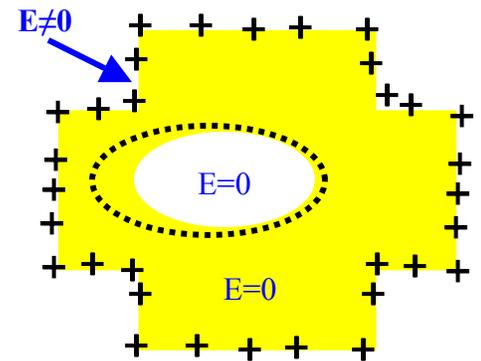
Conductor with net positive charge and a cavity charge.
Cross-section-view.

- **Suppose the total charge on the conductor is $q_{cond} = +2q$. What is the charge on the outer surface?**

  qcond = qinner + qouter
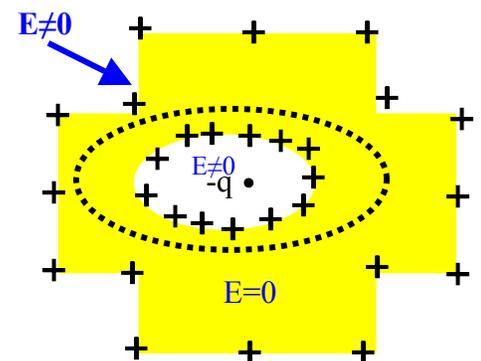
  The charge on the conductor is the sum of the charge on its surfaces.

  qouter = qcond - qinner = +2q - q = +q

After the cavity charge pulled in +q to the inner surface, only +q was left on the outer surface.

Charged Sheets:

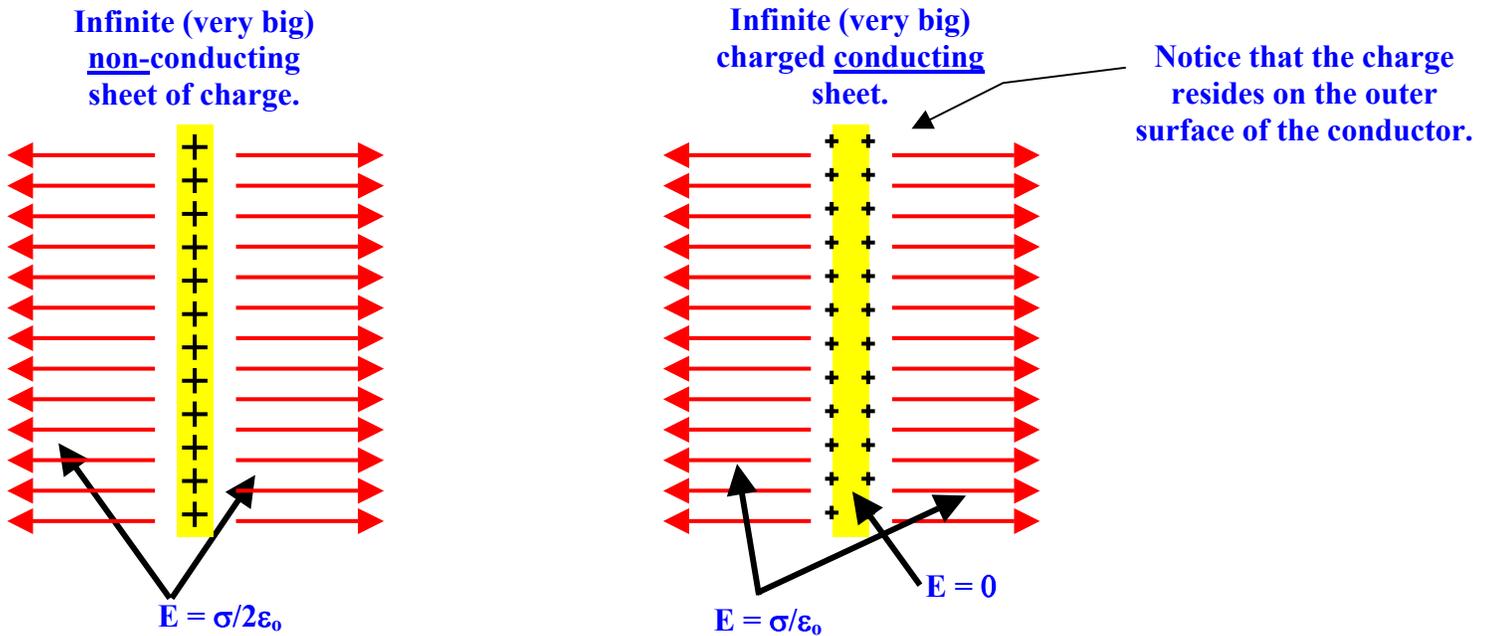**Infinite (very big)
<u>non</u>-conducting
sheet of charge.**

**Infinite (very big)
charged <u>conducting</u>
sheet.**

**Notice that the charge
resides on the outer
surface of the conductor.**

$E = \sigma/2\varepsilon_0$
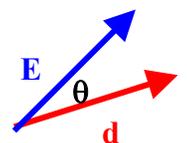
$E = 0$

$E = \sigma/\varepsilon_0$

Electric Potential Energy:

A positive charge loses electric potential energy if it moves down a field line because its motion is in the direction of the electric force.

E

q

$\Delta U < 0$

For a <u>point charge</u> in a <u>uniform field</u> we have: $\Delta U = -q\,\vec{E}\bullet\vec{d}$ **or** $\Delta U = -q\,Ed\cos\theta$. **(This is an important special case!!!)**

E

θ

d

θ is the angle between $\vec{E}$ and $\vec{d}$ when placed tail-to-tail.

Be careful of the negative signs. Note that q can be negative and so can cosθ.

Previously, we said that a charge generates an electric field (a vector field) and that this field can then exert a force on another charge.

Associated with the electric field is a scalar field called the potential. The potential controls how a system (another charge, perhaps) acquires potential energy.

The potential difference between two points in space can be measured using the potential energy change of a point charge between those same two points: $\Delta V = \Delta U / q$. The unit of potential is the volt (V).
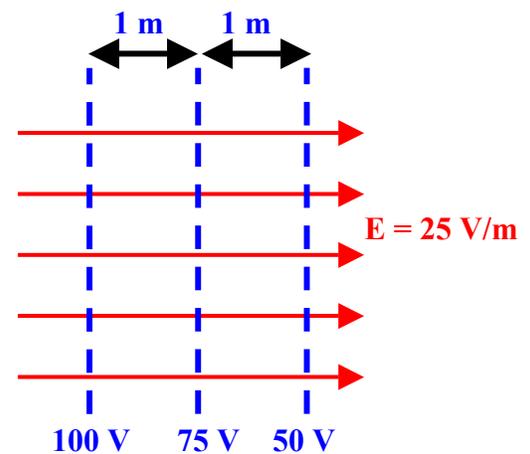
The potential difference between two locations in space in a uniform electric field is: $\Delta V = -\vec{E} \bullet \vec{d}$.
This means the unit of electric field is also V/m.

In general, electric potential decreases as you go down a field line.

We connect points together that are at the same potential (have the same voltage) to form equipotential lines and surfaces.

Equipotential lines are always perpendicular to the electric field lines.

1 m   1 m

E = 25 V/m

100 V   75 V   50 V

In the figure, showing a underline uniform field, note how potential decreases as you go to the right. The potential doesn't change if you go up or down. Also, notice that since E = 25 V/m, you drop 25 volts for each meter traveled to the right.

For a <u>uniform electric field</u>, we have:

$$E_x = -\frac{\Delta V}{\Delta x} \qquad E_y = -\frac{\Delta V}{\Delta y} \qquad E_z = -\frac{\Delta V}{\Delta z}$$

So, for example, given two locations along the x-axis we can determine the x-component of the electric field (in volts/meter) by measuring the potential difference between those two locations (in volts) and the distance between them (in meters).

If the electric field is not uniform, we must use:
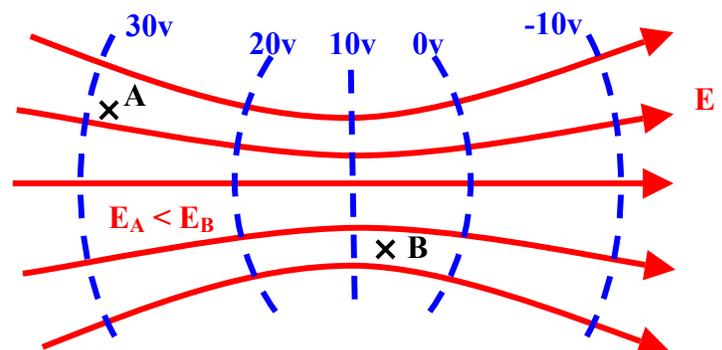
$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

From these relations we conclude that a non-zero electric field *requires the electric potential to vary spatially.*

We can use equipotentials to visualize the variation of potential in space. An equipotential line (or surface) connects points in space that have the same potential.

- **Adjacent equipotentials should always differ by the same voltage difference (10 volts, for example).**

- **Equipotential lines are spaced more closely together where the electric field is stronger *AND* they are always perpendicular to the field lines.**

- **<u>Conductors are equipotentials</u> *THUS* electric field lines near a conductor are always perpendicular to its surface!!!**
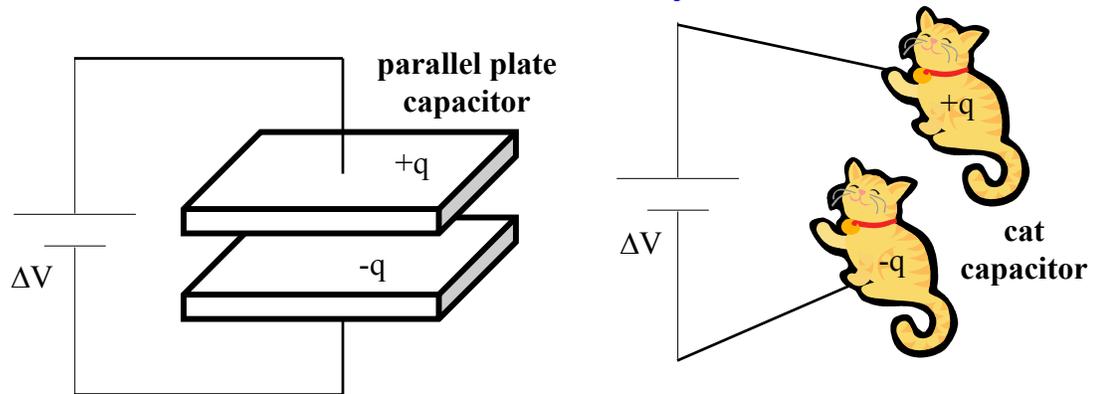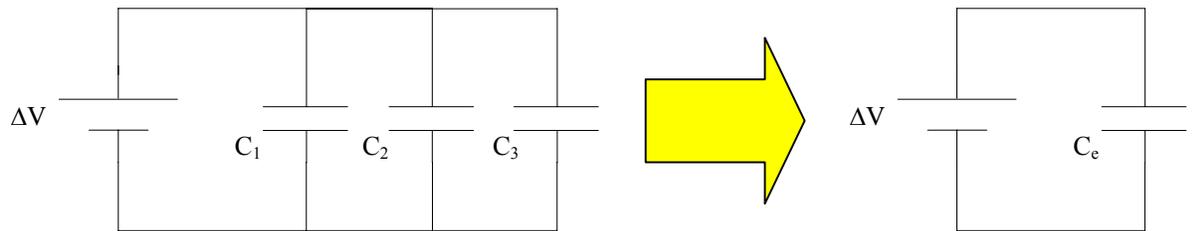
A battery can be used to "pump" charge from one conductor to another. A pair of conductors used in this way is called a capacitor.

The charge q on either plate is given by:

q = C ΔV

parallel plate
capacitor

+q

-q

ΔV

cat
capacitor

+q

-q

ΔV

Capacitors in _parallel_. ΔV
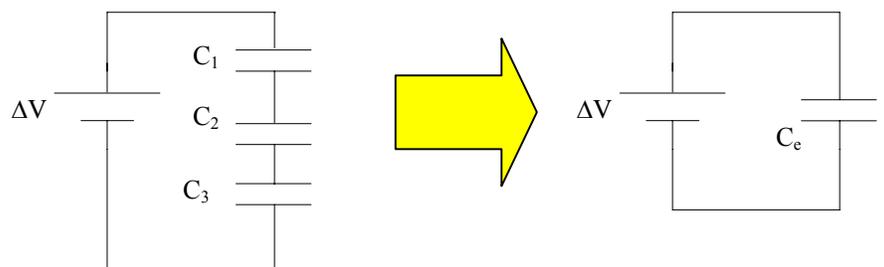
C_1 C_2 C_3

ΔV C_e

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3$$ **True for any set of components in parallel !!!**

$$q_e = q_1 + q_2 + q_3$$ $$C_e = C_1 + C_2 + C_3$$

Capacitors in _series_.

ΔV C_1
       C_2
       C_3

ΔV C_e

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$ **True for any set of components in series !!!**

$$q_e = q_1 = q_2 = q_3$$ $$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

We haven't derived this yet.

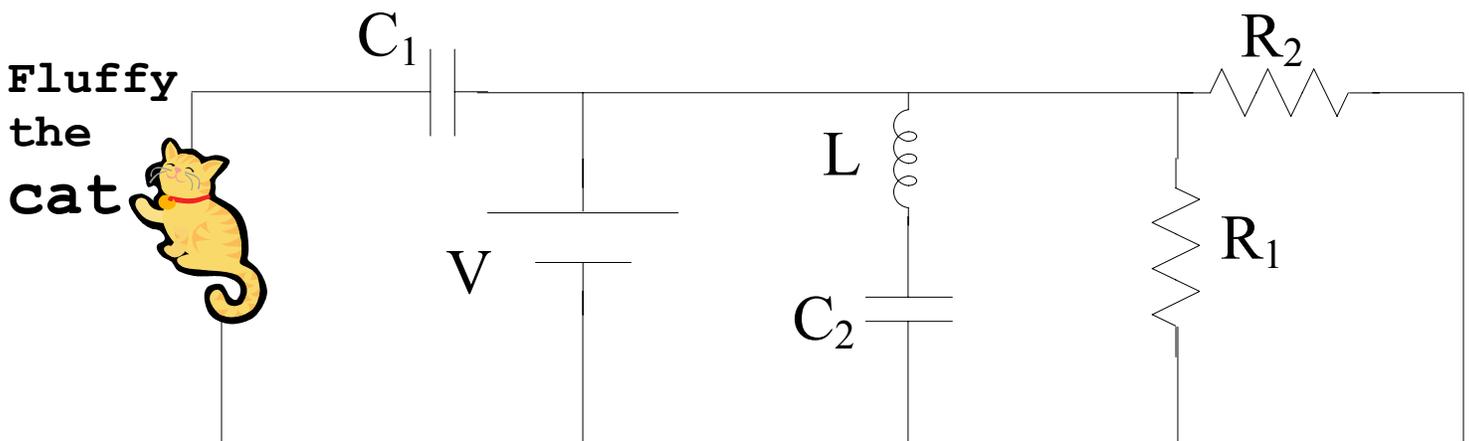**Components in parallel <u>have</u> to have the same voltage.**

Components in series <u>have</u> to share the same current.

Comments

The current (flow of charge) onto or off a capacitor is zero at steady state, but while the current is charging there is a current so you can follow the current flow to help you determine if two caps are in series.

Notice that the two statements above apply to any component, not just capacitors.

A third option is that two components will neither be in series or in parallel. This is usually the case.

Inductor L and capacitor C_2 are in series. (Don't know what an inductor is yet? Doesn't matter. It's still in series with C_2!)

The battery and resistors R_1 and R_2 are all in parallel.

C_1 and Fluffy are in series.

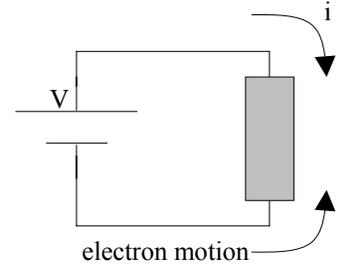**C_1 and R_2 are neither in series or parallel.**

CURRENT

The electric current is the amount of charge passing through a reference plane per unit time:
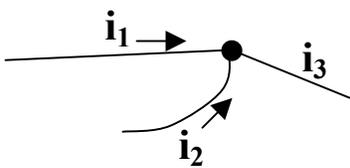
$$i = dq/dt \quad \text{or, for a constant current,} \quad i = \Delta q/\Delta t.$$

The unit of current is the <u>C/s or Amp (A)</u>.

The current direction is *defined* as the direction that a positive charge would travel. For circuits, the current is in the opposite direction of the electron motion.

electron motion

Node rule: $i_{in} = i_{out}$ at a node,   OR
The sum of the currents at a node is zero.

$i_1 + i_2 + i_3 = 0.$

So, if $i_1 = 2A$ and $i_2 = 3A$, $i_3 = -5A.$

(The minus means the current is going out of the node.)

RESISTANCE

The electric field inside a conductor when its charges are at rest (electrostatic case) is zero. When a current flows through a conductor, there <u>*is*</u> an electric field, given by: $\vec{E} = \rho \vec{J}$

$\vec{J}$ **is the current density. If the current is uniform: $J = i/A$.**

The direction of $\vec{J}$ is the same as the current. ρ is the resistivity.

It is a <u>property of the medium</u> and has units $(V/A)\ m = \Omega\ m$.

RESISTORS

These are components designed to resist the flow of current. They are characterized by a "resistance" such that $R = V/i$ (in Ω) or $V = iR$.

POWER

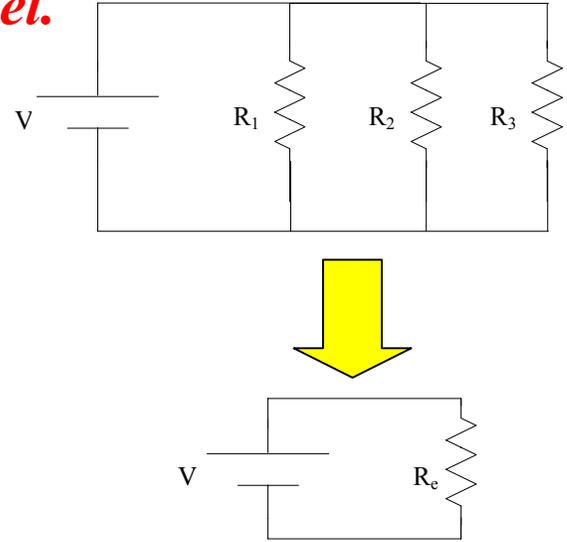**$P = iV$ (in general!) $P_R = i^2R = V^2/R$ (for resistors)**

Recap: Lecture #17 Multiloop Circuits

Resistors in combination: Parallel.

$$i_e = i_1 + i_2 + i_3$$

$$V_e = V_1 = V_2 = V_3$$

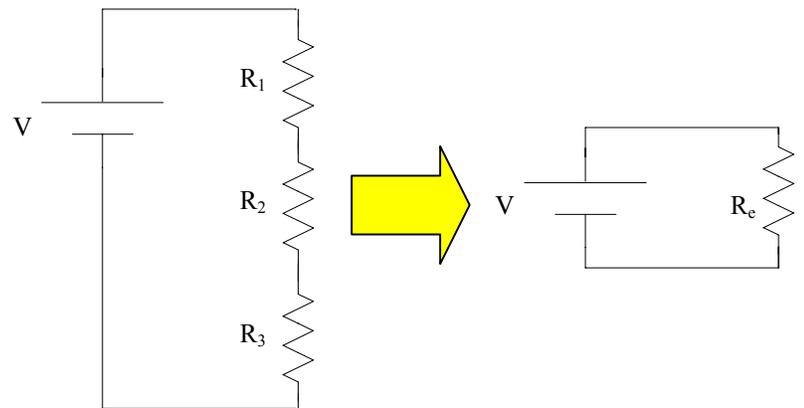$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

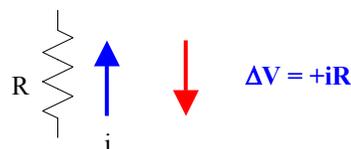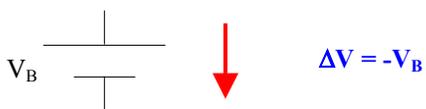*Resistors in combination: Series.*

$$i_e = i_1 = i_2 = i_3$$

$$V_e = V_1 + V_2 + V_3$$

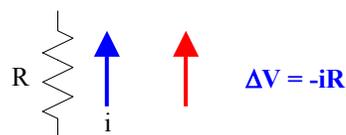$$R_e = R_1 + R_2 + R_3$$

Various components in any order: node and loop rules.

To use the loop rule, you need to know how to determine potential differences:

V_B $\Delta V = +V_B$

R i $\Delta V = -iR$

V_B $\Delta V = -V_B$

R i $\Delta V = +iR$

Red arrows indicate your
step direction as you "walk"
around a loop.

Blue arrows indicate the
current direction.

Recap: Lecture #18   Problem solving

We looked at this circuit and started by identifying the currents. <u>Currents go between adjacent nodes.</u> Some are identified in the figure.

$E_1 = 10V$, $E_2 = 20V$
$R_0 = R_1 = R_2 = 100\ \Omega$.
$R_3 = 300\ \Omega$, $R_4 = 400\ \Omega$.

i_4 was found by inspection: $i_4 = E_2/R_4$.

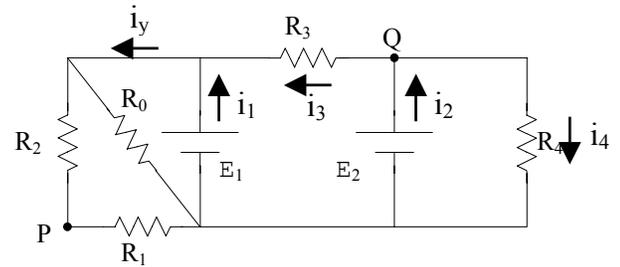**i_3 was found using the loop rule and the loop in blue.**

i_2 was found using the node rule and node Q.

We decided the best way to get i_y was to replace the resistor network on the left with an equivalent resistor: $R_{012} = 67\ \Omega$.
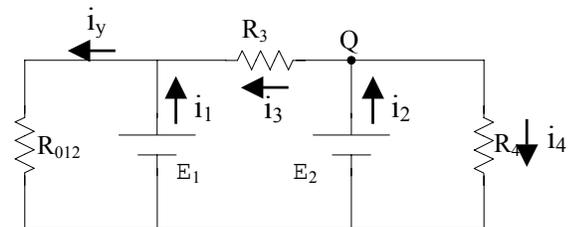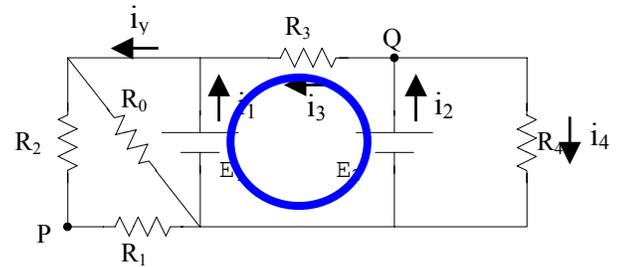
We stopped here, but the remaining currents can be found in the same fashion if desired.

The point of this exercise was to make clear that a variety of strategies must be brought into play to characterize a circuit efficiently.

Here are the solutions to the circuit:

i_4) $i_4 = E_2/R_4 = 1/20$ A.

i_3) $E_2 - i_3R_3 - E_1 = 0$ ▶ $i_3 = (E_2 - E_1)/R_3 = 10V/300\Omega = 1/30$ A.

i_2) $i_2 = i_3 + i_4 = 1/12$ A (using the node above E_2)
  $P_{E2} = E_2\, i_2 = 20/12$ W (battery E_2 is supplying power)

i_y) $i_y = E_1/R_{012} = 10/67$ A $= 0.15$ A.

i_1) $i_1 + i_3 = i_y$ ▶ $i_1 = i_y - i_3 = 0.12$ A.
  $P_{E1} = E_1\, i_1 = 1.2$ W (battery E_1 is supplying power)

$\underline{V_Q - V_P}$

We solve this class of problem by "walking" from location P to Q keeping track of the potential changes found along the way. Any path from P to Q will do, although it is easier to walk over batteries rather than resistors.

I'll use the purple path. We start at point P and thus at potential V_P. Our potential changes as we walk towards Q. When we get to Q, our potential is V_Q:

$$V_P - i_xR_1 + E_2 = V_Q$$

$$V_Q - V_P = E_2 - i_xR_1$$

We need i_x. $i_x = E_1/(R_1 + R_2) = 1/20$ A.

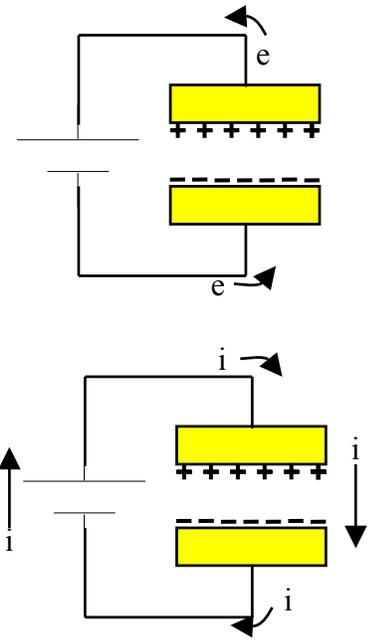$$V_Q - V_P = 20V - (1/20A)(100\Omega) = 20V - 5V = 15V.$$

Location Q is at higher potential than location P.

When a capacitor charges, electrons are pumped from one plate to the other. *Effectively,* **this is no different than a current flowing** *through* **the capacitor in the opposite direction.**

When charging or discharging we will treat capacitors as having a current even though, in reality, no electrons ever jump the gap from one plate to the other.

- **An initially uncharged capacitor effectively acts likes a wire when it first starts to charge because pulling the first electron off is easy.**

At t = 0.

$i = E/R$

$V_c = 0$

- **A fully charged capacitor blocks steady state currents. It acts like a broken wire.**

At t = a long time later

$i = 0$

$V_c = E$
(in this case)

From the demonstrations it appears we must conclude:

- **A non-contact force can exist between currents, and it's not the gravitational or electric force.**

- **A force can exist between a magnet and a current.**

We guessed (correctly, it turns out) that the force between currents is also magnetic.

Currents generate magnetic fields and magnetic fields can exert forces on other currents.

We'll consider the way in which currents generate magnetic fields later. For now, we simply accept that they exist. What force do they exert? Start with the simplest situation: *A moving point charge in a uniform field.*

The magnitude of the force is given by:

$$F = qvB \sin\theta$$

So, if the charge doesn't move, it won't experience a force. You have to have a charge in motion for it to be a current!

If the charge moves parallel to a field line, $\theta = 0$, and again the magnetic force on it is zero.

The magnitude *and* the direction of the force are given in a single equation by: $\vec{F} = q\,\vec{v} \times \vec{B}$

The cross product, $\vec{A} \times \vec{B}$**, is a vector that is perpendicular to both** $\vec{A}$ **and** $\vec{B}$**. Its magnitude is:** $\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$

Right-hand rule for cross-products:

A   A x B $\otimes$

B

A   B x A $\odot$

B

Now, since we have: $\vec{F} = q\,\vec{v} \times \vec{B}$ **we see immediately that:**

- **The force is zero for stationary charges.**
- **The force is zero if the velocity vector is parallel (or anti-parallel) to the field.**
- **The force is perpendicular to both the velocity vector and the field.**
- **The force on a positive charge is opposite in direction to the force on a negative charge.**

Finally, if we have a <u>straight</u> wire carrying a current, the force on the wire due to a magnetic field is: $\vec{F} = i\,\vec{L} \times \vec{B}$.

$\vec{L}$ **is a vector that describes the wire: L is the length of the wire that is in the magnetic field and the direction is the same as the current.**

B

i

F

L

RHR for currents generating a magnetic field!!!!!
Magnetic field lines <u>circle around currents</u>. Grasping
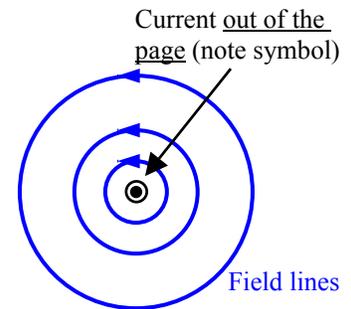**the current with your right hand and your thumb in the**
direction of the current, your fingers give the direction
of the field lines.

Current <u>out of the</u>
page (note symbol)

Field lines

Ampere's Law: $\oint \vec{B} \bullet d\vec{l} = \mu_0 i_{enc}$.

You <u>need</u> to pick a closed path (a loop). Different parts of
the path may contribute differently to the integral.

(I)	**If $\vec{B}$ is perpendicular to a part of the path, then for that part:** $\int \vec{B} \bullet d\vec{l} = 0$
(IIa)	**If $\vec{B}$ is constant and parallel to a part of the path, then for that part:** $\int \vec{B} \bullet d\vec{l} = BL$ **(where L = the path length).**
(IIb)	**If $\vec{B}$ is constant and anti-parallel to a part of the path, then for that part:** $\int \vec{B} \bullet d\vec{l} = -BL$.

<u>The thin infinite wire.</u>

According to case IIa:

$\int \vec{B} \bullet d\vec{l} = BL = B(2\pi r)$

Now we need i_{enc}:

$i_{enc} = i$

Finally:

i, into
the page

R

Amperian loop in red
(just follows the field
line, for this problem).

r

Field line
in blue.

Notice you have to give
your path, or Amperian
loop, dimensions. In this
case, a radius r.

$B(2\pi r) = \mu_0 i_{enc} = \mu_0 i$ **so, we get:** $B = \dfrac{\mu_0 i}{2\pi r}$

An infinitesimal piece of wire $d\vec{l}$ will generate a magnetic field $\vec{B}$ at a location selected by $\vec{r}$ according to the Law of Biot and Savart: $d\vec{B} = \dfrac{\mu_0 i}{4\pi} \dfrac{d\vec{l} \times \vec{r}}{r^3}$

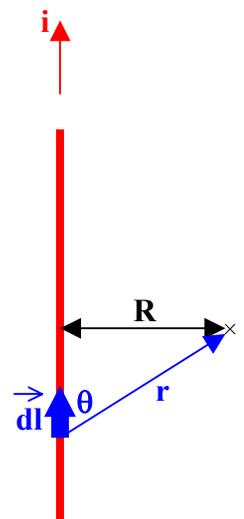**So, $d\vec{B}$ has magnitude** $dB = \dfrac{\mu_0 i}{4\pi} \dfrac{dl \sin\theta}{r^2}$ **and direction given by the RHR: Into-the-page, for the case shown.**

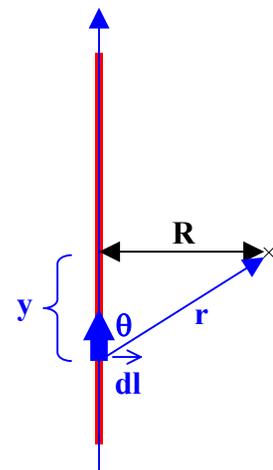**Using the Law of Biot and Savart to find the magnetic field from a long straight wire:**

(1) Pick a small piece of the wire to be your $d\vec{l}$.

(2) Add the vector $\vec{r}$ from $d\vec{l}$ to the location where you want to find the field.

(3) The direction of $d\vec{B}$ is given by $d\vec{l} \times \vec{r}$, into-the-page in this case.

(4) For each component of the field (only one in this case), construct an integral: $B = \dfrac{\mu_0 i}{4\pi} \displaystyle\int \dfrac{\sin\theta}{r^2} dl$

(5) Introduce a variable (or coordinate) that indicates where the wire-piece is located.

$dl = dy$ $r = \sqrt{y^2 + R^2}$ $\sin\theta = R / r$

(6) Solve the integral.

$$B = \dfrac{\mu_0 i}{4\pi} \int\limits_{-\infty}^{+\infty} \dfrac{R}{\left(y^2 + R^2\right)^{3/2}} dy = \dfrac{\mu_0 i R}{4\pi} \dfrac{2}{R^2} = \dfrac{\mu_0 i}{2\pi R}$$

<u>Can crusher:</u> The magnetic field generated by the solenoid shouldn't be able to exert a force on the can because there shouldn't be any currents in the can. It's not connected to anything. However, the can is crushed. We know from our study of the electric motor that <u>if</u> there were a loop current in the can somehow, that would explain its being crushed.

Hypothesis: We have a current in the can or there exists still yet another new force or something even crazier.

<u>Ring toss:</u> Again we get a force on a conductor that "shouldn't" have any currents in it. To test whether we have a current or a new force, we can try a split ring. The gap in the ring will prevent currents from circulating around the ring. A new force would presumably still act on the conductor. Result: We find that the force on the ring goes away. Kill the possibility of a current and you kill the force.

Hypothesis: A current is generated in the ring and the magnetic field present exerts a force on it.

Why is a current generated? Well, in all this we are doing one thing different from everything else we've seen so far. We are using a time-varying magnetic field.

Hypothesis: A time-varying magnetic field can generate a current in a conductor and then exert a force on that current. (We call the generation process "induction".)

Faraday's Law tells us how to make a power supply (or EMF source) using magnetic fields. Almost all electric power generation in the world is done via the use of Faraday's Law.
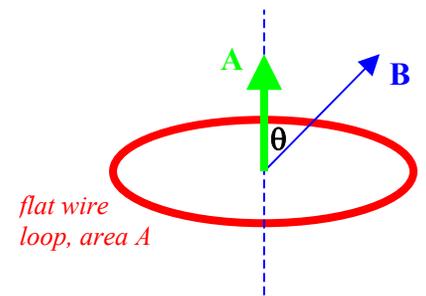
Faraday's Law: $E = -N \dfrac{d\Phi_B}{dt}$

E = the EMF generated throughout a loop. (I haven't discussed why we use the phrase "EMF" yet instead of something like "voltage difference".)

Φ_B = the magnetic flux going through the surface defined by the loop. For purposes of this class, we are only concerned about the flux due to an *external* magnetic field, not any flux generated by the loop itself.

N = the number of turns in the loop (for example, you might wrap a wire into a circular loop going around N=100 times).

Let's limit discussion for now to the special case of *flat loops in uniform fields*. Then:

$$\Phi_B = \vec{B} \bullet \vec{A} = BA\cos\theta$$

So, to use Faraday's Law just calculate the magnetic flux and then take the time derivative. (I'll discuss the minus sign later.)

flat wire loop, area A

There are three basic possibilities for the derivative. More complicated possibilities including multiple time dependencies and more complicated surfaces and fields can be readily handled by extension.

B = B(t), A and θ constant: $\dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(BA\cos\theta\right) = A\cos\theta \dfrac{dB}{dt}$

A = A(t), B and θ constant: $\dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(BA\cos\theta\right) = B\cos\theta \dfrac{dA}{dt}$

θ = θ(t), A and B constant: $\dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}\left(BA\cos\theta\right) = BA\dfrac{d\cos\theta}{dt} = BA\left(-\sin\theta \dfrac{d\theta}{dt}\right)$

This last result requires the chain-rule in the last step. To go further, you will need θ(t). For the case where the loop rotates uniformly with period T, (or frequency f = 1/T) we have: θ(t) = 2π t/T = 2πf t,   θ in radians.
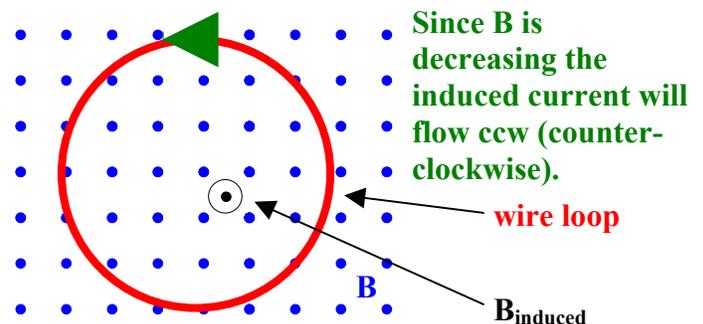
For the problems we'll look at, use Faraday's Law to get the magnitude of the EMF. Use Lenz's Law to determine which way the EMF is oriented (or which way it will try to drive a current).

Lenz's Law: The EMF generated will always try to drive a current that <u>opposes</u> the change in flux. Lenz's Law is a consequence of the conservation of energy.

Example. The flux through the loop is out-of-the-page and decreasing. We know there will be an induced current, but cw or ccw?

B is decreasing

wire loop

B

Well, the induced current will generate it's own magnetic field *in addition* to the external field B. Let's call this new field B_{ind}. From Lenz's Law it must be oriented to oppose the change in

Since B is decreasing the induced current will flow ccw (counter-clockwise).

wire loop

B

$B_{induced}$

flux. A decreasing flux out-of-the-page will be opposed (partially compensated for) if B_{ind} is also out-of-the-page. The only way B_{ind} can be out-of-the-page is if the current flows CCW.

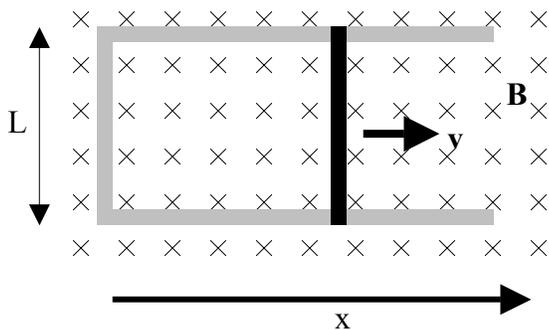**For the problems we'll consider, B_{ind} is significantly smaller than B and is never able to prevent the flux from changing. You won't need to worry about the force B_{ind} will exert or anything else. That doesn't mean it's unimportant but, for us, it is mainly a thought-tool that helps us apply Lenz's Law.**

Recap: Lecture #27   The Rail Problem

Mathematically, this problem isn't hard. However, you have to be able to apply Lenz's Law, use a number of different right-hand-rules, keep the various angles that come into play straight, etc. Circuit concepts and principles from P131 have to be recognized, as well. The challenge isn't math then, but to put _everything_ together. The rest of class will be like this! Here it is again:

The rail has mass M and resistance R and is moving at constant speed v.

B = 1T, R = 10 Ω, L = 10 cm, v = 10 m/s.

Find: E (the loop EMF), i (the loop current), $\vec{F}_{ext}$ (the external force maintaining the motion).

Inside the loop formed by the rails and rod, we have an increasing flux into-the-page so B$_{ind}$ will be out-of-the page. This requires that the induced current flow ccw (counter-clockwise). Thus, the induced current flows upwards through the rod. Because of this current, there will be a magnetic force on the rod to the left. F$_{ext}$ must cancel this force since the rod moves at constant speed.

$$E = N\frac{d\Phi_B}{dt} = \frac{d}{dt}(AB\cos\theta) = B\frac{dA}{dt}$$

Comments: N = 1 because the loop only has 1 turn. B is a constant and comes out of the derivative. θ is the angle between the normal to the loop surface and the field and is either 0° or 180° for our problem, so the cosine is either 1 or -1. (We only want the magnitude of E at this point, so we don't care about signs.)

To take the derivative, I need an expression for the area. Let x be the distance the rail has traveled. Then A = Lx (length times width).

$$\frac{dA}{dt} = \frac{d}{dt}(Lx) = L\frac{dx}{dt} = Lv$$

The rest follows:

$$E = BLv \qquad i = E/R = BLv/R$$

$$F_{ext} = F_B = iLB\sin\theta = (BLv/R)LB\sin 90^o = B^2L^2v/R$$

Put in the numbers to see what's possible.

This comes from $\vec{F} = i\vec{L}\times\vec{B}$. So here "θ" is the angle between $\vec{L}$ (the current direction in the rod) and $\vec{B}$.