Problem 1. Two lenses are arranged as shown. An arrow pointing upwards is placed before the converging lens. The total system magnification is \( m = 0.418 \). What is \( f_2 \)?

Use, \( p_1 = 300 \text{ mm}, \quad f_1 = +500 \text{ mm}, \quad d = 250 \text{ mm}. \)

\[
\begin{align*}
\frac{L_1}{f_1} &= \frac{p_1 f_1}{p_1 - f_1} = \frac{(300 \text{ mm})(500 \text{ mm})}{300 \text{ mm} - 500 \text{ mm}} = -750 \text{ mm} \quad \text{(virtual)} \\
M_1 &= -\frac{L_1}{p_1} = -\frac{-750 \text{ mm}}{300 \text{ mm}} = 2.5 \quad \text{(not inverted)} \\
\end{align*}
\]

\[
\begin{align*}
\frac{1}{f_2} &= \frac{R_1}{R_1 + L_2} \\
R_2 &= |L_1| + d = 750 \text{ mm} + 250 \text{ mm} = 1000 \text{ mm} \\
M &= M_1 M_2 \\
M_2 &= \frac{M}{M_1} = \frac{0.418}{2.5} = 0.167 \quad \text{(not inverted)} \\
\end{align*}
\]

We are left with two equations in two unknowns: \( L_2 \) and \( f_2 \).

\[
\begin{align*}
L_2 &= -R_2 M_2 \\
f_2 &= \frac{R_2 (-R_2 M_2)}{R_2 + (-R_2 M_2)} = -\frac{R_2^2 M_2}{R_2 (1 - M)} = \frac{R_2^2 M_2}{R_2 (M-1)} \quad \checkmark \\
= \frac{(1000 \text{ mm})^2 (0.167)}{(1000 \text{ mm})(0.167-1)} &= -200 \text{ mm} \quad \checkmark
\end{align*}
\]
**SAMPLE SECOND MIDTERM**

**Problem 2.** A beam of light, \( \lambda = 500 \) nm, enters a right triangle made of glass at normal incidence. The surrounding medium is air. Lengths and angles are as shown in the figure with \( D = 1.0 \) cm.

(a) Some of the light will always exit through side B. Depending on the index of refraction, some may also exit through side A. Sketch the path the beam will take for both of these cases. Draw the normal to the surface of the prism where each beam exits.

(b) What is the speed of the wave in the prism if its index of refraction is the smallest value that results in no light exiting through side A?

\[
V = \frac{c}{n} \text{ so we need } n.
\]

\[
\sin \theta_c = \frac{n_{\text{prism}}}{n_{\text{air}}} = \frac{n_{\text{prism}}}{n} = \frac{1}{n}
\]

\[
n = \frac{1}{\sin \theta_c} = \frac{1}{\sin 45^\circ} = 1.414
\]

\[
V = \frac{3 \cdot 10^8 \text{ m/s}}{1.414}
\]

(c) What is the frequency of the light inside the prism?

Some vs in air

\[
f = \frac{c}{\lambda} = \frac{3 \cdot 10^8 \text{ m/s}}{500 \cdot 10^{-9} \text{ m}} = 6.0 \cdot 10^{14} \text{ Hz}
\]

(d) Starting from the entrance, how far inside the prism must you go for the wave to have a phase difference of \( 2\pi \) with respect to the wave entering the prism?

\[
\Delta \phi = 2\pi \frac{\Delta L}{\lambda_{\text{prism}}}
\]

\[
\Delta L = \frac{\Delta \phi}{2\pi} \frac{\lambda_{\text{prism}}}{n} = \frac{\Delta \phi}{2\pi} \frac{\lambda}{n}
\]

\[
= \frac{2\pi}{2\pi} \frac{500 \cdot 10^{-9}}{1.414} = 354 \text{ nm}
\]
Problem 3. In a Young's Two Slit Interference apparatus, the spacing between the slits, $d$, is 0.15 mm. The interference pattern from it is viewed a distance $D = 1.0$ m from the slits.

(a) What is the phase difference between waves from the two slits that arrive at the second minimum?

$$\Delta \phi = (m + \frac{1}{2}) \frac{2\pi}{d} = \frac{3}{2} \frac{2\pi}{d} = 3\pi \sqrt{\frac{d}{m}}$$

$m = 1$

(b) What is the separation between the point of greatest intensity when both slits are used, and the point of greatest intensity when the bottom slit is blocked?

$$d_{AB} = 0.075 \text{ mm}$$

(c) Suppose the slits are illuminated with two different wavelengths: $\lambda_A = 500$ and $\lambda_B$. It is observed that the second minimum when using $\lambda_A$ occurs in the same place as the second maximum when using $\lambda_B$. What is $\lambda_B$?

$m_A = 1$

$$d \sin \theta_A = (m_A + \frac{1}{2}) \lambda_A$$

[footnote: for constructive interference & $A$]

$$d \sin \theta_B = m_B \lambda_B$$

["" destructive """]

but $\theta_A = \theta_B$ so

$$(m_A + \frac{1}{2}) \lambda_A = m_B \lambda_B$$

$$\frac{3}{2} \lambda_A = \lambda_B$$

$$\lambda_B = \frac{2}{3} (500 \text{ nm}) = 750 \text{ nm} \sqrt{\frac{d}{m}}$$
Problem 4.

(1) A light beam passes from plastic to glass. As it does so, it bends away from the normal to the interface between them. We can conclude:

(a) $n_{\text{plastic}} > n_{\text{glass}}$
(b) $n_{\text{plastic}} = n_{\text{glass}}$
(c) $n_{\text{plastic}} < n_{\text{glass}}$
(d) $n_{\text{plastic}} = n_{\text{air}}$
(e) $n_{\text{glass}} = n_{\text{air}}$

(2) The graph on the right gives the value of the electric field in an EM wave as a function of position at time $t = 0$. Which graph below best describes the magnetic field as a function of time at $x = 0$? The wave is moving in the -$x$ direction.

(a) $E(t)$  (b) $E(t)$  (c) $B(t)$  (d) $B(t)$

\textit{Graphs of $E(x)$ and $B(x)$ look the same. Likewise $E(x)$, $B(x)$}

(3) An EM wave travels in the -$x$ direction. Its electric field has maximum magnitude, $E_m$. A possible expression for the magnetic field is:

(a) $\vec{B} = \vec{x} \frac{E_m}{c} \sin(kx - \omega t + \phi_e)$
(b) $\vec{B} = \vec{x} cE_m \sin(kx - \omega t + \phi_m)$
(c) $\vec{B} = \vec{y} \frac{E_m}{c} \sin(kx - \omega t + \phi_e)$
(d) $\vec{B} = \vec{y} cE_m \sin(kx - \omega t + \phi_m)$

\textit{Points along $\vec{B}$ or $\vec{Z}$}

(4) Let the wavelengths of red light and blue light be $\lambda_R$ and $\lambda_B$, respectively. Then,

(a) $\lambda_R < \lambda_B$
(b) $\lambda_R = \lambda_B$ (only their frequencies differ).
(c) $\lambda_R > \lambda_B$.

(5) The $m = 2$ bright fringe from a two-slit interference apparatus is observed before and after the apparatus is modified. Which of the following will change the phase difference between waves arriving at the $m = 2$ bright fringe?

(a) immersing the apparatus in water
(b) increasing the spacing between slits
(c) both of the above
(d) neither of the above

\textit{By definition, the $m = 2$ bright fringe has $\Delta \Phi = 4\pi$}