Practice and review problems for the first physics 570 midterm.

As stated in class, these problems do not constitute a practice midterm. For example, they are not designed to be solved within some time limit. Also, they don't cover all topics that we've treated. They are provided to help you gauge your preparation. A well prepared student should be able to solve all of these.

Rules of thumb and basic facts.

The midterm problems similar to these will be given as a separate component to be taken without notes or the text. You should have a calculator (with cleared memory). You will only be given a few minutes on the actual test.

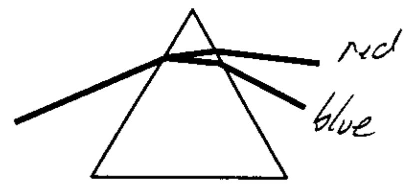
(a) What is the mass of an electron (in any units you choose, 2 significant figures or more)?

(b) What are Maxwell's equations (general form without use of D, H, ε, μ)?

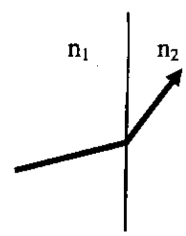
(c) What is a typical energy for an optical photon (within a factor of 2)?

(d) Is 250 nm light in the IR, optical, or UV spectrum?

(e) Red and blue laser beams that are initially coincident enter a prism as shown. Which of the exiting beams is the blue one?



(f) A ray of light is shown crossing the interface between two media in the figure. Which medium has the larger index of refraction?



(a) $mc^2 = 511$ KeV

(b) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ $\vec{\nabla}\cdot\vec{B} = 0$ $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

(c) ~ 2 eV

(d) UV

(e) as shown

(f) n_1

General problems.

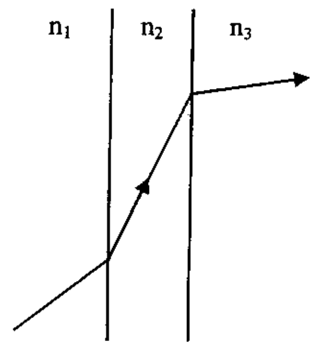
Open notes (mine and yours if taken in class), homework solutions, text, calculator.

(1) An electromagnetic wave is propagating through space such that:

$$\vec{E} = (3500\,\mathrm{V/m})\,\hat{j}\ \sin\left[(250\,\mathrm{m}^{-1})\,x - (100\,\mathrm{m}^{-1})y + \omega t + 3\pi/2\right].$$

Find: The propagation direction specified as an angle with respect to the x-axis; the direction of $\vec{B}$ at x=0, t=0; the maximum magnitude of $\vec{B}$; and the frequency.

(2) The maximum magnitude of the electric field 30 cm from a point source of light is 100 V/m. What is the maximum magnitude of the electric field 120 cm from the point source?



(3) A ray of light travels through three media as shown. Rank the three indices of refraction, n_1, n_2, and n_3 from smallest to largest.

(4) A point source of light is 2.20 m below the surface of a pool containing a liquid with n= 1.83. Some of the light will leave the pool and enter the air and some will be reflected at the surface. Find the diameter of the circle at the surface through which the light emerges from the pool into air.

(5) Show that $f(x,t) = a(x + vt)^b$ where a and b are real constants and v is the wave speed satisfies the 1-D wave equation.

(6) Light is normally incident on a flat metal block with $\omega_p = 1.3 \times 10^{16}$ rad/s. The reflectivity is measured as a function of decreasing wavelength. Below a certain value, the reflectivity suddenly drops. What is this value?

(7) The solar intensity near the earth is approximately 1.3 kW/m^2. How large would an 85% reflecting solar sail have to be to give a 500 kg vehicle an acceleration of 10^{-2} g? Assume the light is normally incident.

(8) A 100 W/m^2, 488 nm, s-polarized laser is internally incident on a plastic/air interface at an angle exceeding the critical angle by 10°. $n_{plastic} = 1.44$. **(a)** What is the peak electric field of the incident light before the interface? **(b)** What is the peak electric field magnitude 100 nm from the interface on the air side?

(10) What is the refractive index of a glass plate that polarizes light reflected at 57.5°?

(11) Derive an expression for the transmitted magnetic field after total internal reflection for p-polarized light.

(12) In some medium, 532 nm light travels at 2.3×10^8 m/s and the intensity decreases by half for every 1.0 mm of travel. **(a)** What is the complex index of refraction? **(b)** Qualitatively, how would the index of refraction change if 535 nm light is used?

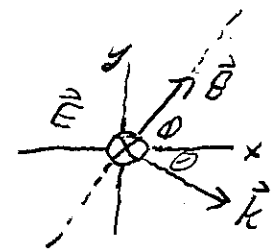
(1) $k_x = 250$ rad/m $k_y = -100$ rad/m $k_z = 0$ thus:

the propagation direction is in the xy-plane at an angle wrt the x-axis $\theta = \tan^{-1}\left(\frac{k_y}{k_x}\right) = -21.8^\circ$ ✓

$k = \sqrt{k_x^2 + k_y^2}$ $\omega = kv = kc$ $f = \frac{\omega}{2\pi} = 12.9$ GHz ✓

$E_0 = 3500$ V/m $B_0 = \frac{E_0}{c} = 11.7\ \mu T$ ✓

At $\vec{r} = t = 0$, $\vec{E} = E_0 \hat{k} \sin\frac{3\pi}{2} = -E_0 \hat{k}$



Since $\vec{B} \perp \vec{E}$, it lies in the xy plane.

Since $\vec{B} \perp \vec{k}$, it lies on the dotted line.

By right-hand-rule, it must be as shown with $\phi = 90^\circ - \theta = 68.2^\circ$ ✓

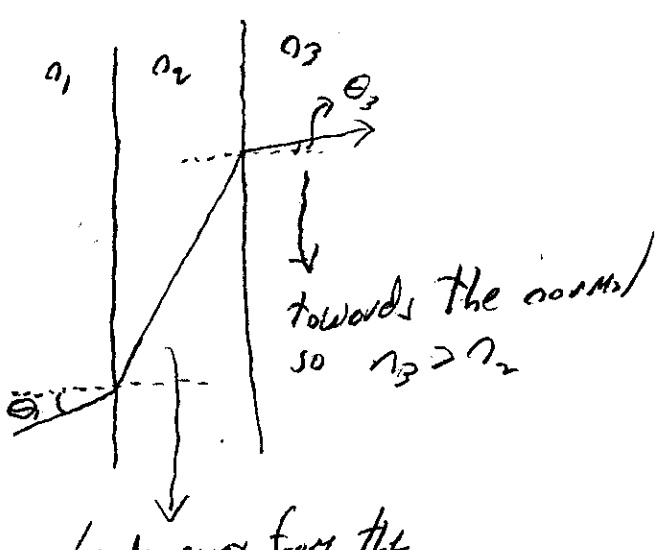
(2) Point source emits spherical waves so $I(r) \propto \frac{1}{r^2}$

thus $E(r) \propto \frac{1}{r}$ ⇨ $\frac{E_1}{E_2} = \frac{r_2}{r_1}$ ⇨ $E_{20} = E_{10}\frac{r_1}{r_2}$

$= \left(100\ \frac{V}{m}\right)\left(\frac{30\text{ cm}}{120\text{ cm}}\right)$

$= 25$ V/m ✓

(3)

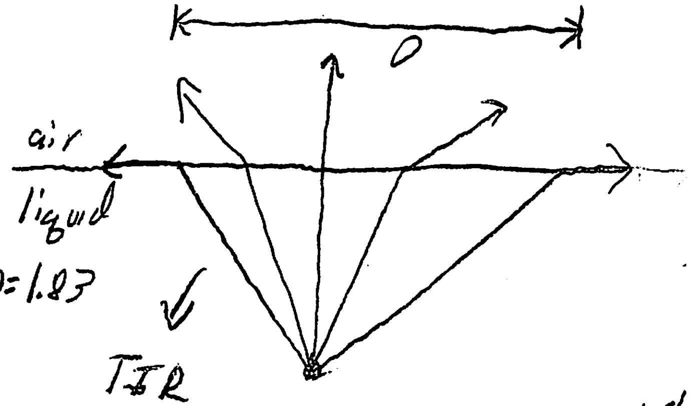


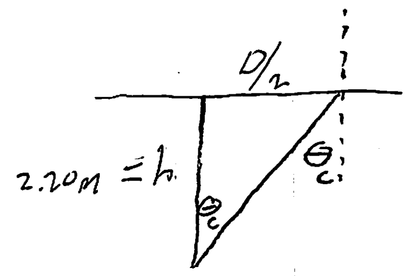
If we had $n_1 = n_3$, then $\theta_1 = \theta_3$.
Clearly, $\theta_3 < \theta_1$ so

$n_3 > n_1$.

$n_3 > n_1 > n_2$ ✓

(4)





$$\theta_c = \sin^{-1}\left(\frac{1}{1.83}\right) = 33.1°$$

$$\tan\theta_c = \frac{D/2}{h} \Rightarrow D = 2h\tan\theta_c = 2.87\,m \checkmark$$

(5) $\frac{\partial^2 f}{\partial x} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$$\frac{\partial f}{\partial x} = ab(x+vt)^{b-1}$$

$$\frac{\partial^2 f}{\partial x^2} = ab(b-1)(x+vt)^{b-2}$$

$$\frac{\partial f}{\partial t} = abv(x+vt)^{b-1}$$

$$\frac{\partial^2 f}{\partial t^2} = ab(b-1)v^2(x+vt)^{b-2}$$

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial x^2} = ab(b-1)(x+vt)^{b-2}$$

✓

(6) Decreasing λ means increasing ω.
At $\omega > \omega_p$, the EM wave begins to transmit.

$$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} = \frac{2\pi c}{\omega_p} = 145\ nm \checkmark$$

7. $F = ma$

$m = 500\ kg$

$a = 9.8 \cdot 10^{-2}\ m/s^2$

$I_{sun} = 1300\ W/m^2$

$PA = ma$

$P = \frac{I_{sun}}{c}\left(0.15 + 2 \times 0.85\right)$

$= 8.1 \cdot 10^{-6}\ Pa$

$A = \frac{ma}{P} = 6.1 \cdot 10^{6}\ m^2$ (roughly a square $2\frac{1}{2}$ km on a side)

8. $\sin\theta_c = \frac{1}{n_{plastic}} \Rightarrow \theta_c = 44.0^\circ$

$\theta_i = \theta_c + 10^\circ = 54.0^\circ$

$\alpha = k_t \left[\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1\right]^{1/2}$

$= \frac{2\pi}{488\ nm}\left[\left(\frac{1.44}{1}\sin 54^\circ\right)^2 - 1\right]^{1/2}$

$= 0.00770\ nm^{-1}$

9. $I = \frac{\varepsilon_0 c n E_0^2}{2} = 100\ W/m^2$

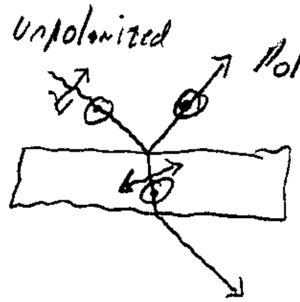
$E_0 = \left(\frac{2I}{\varepsilon_0 c n}\right)^{1/2} = 229\ V/m$ ✓

b) $E_{0,gap} = E_0 e^{-\alpha z} = E_0 e^{-(0.00770\ nm^{-1})(100\ nm)}$

$= E_0 (0.463)$

$= 106\ V/m$ ✓

(9)



Polarized if $\theta_i = \theta_B$

$$\tan\theta_B = \frac{n_t}{n_i}$$

$$n_t = n_i \tan\theta_B = n_i \tan\theta_i = (1.0)\tan 57.5^\circ$$
$$= 1.57 \checkmark$$

(10)

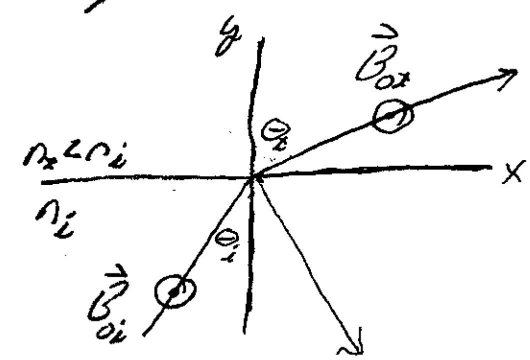
For $\theta_i < \theta_c$,

$$\vec{B}_t = B_{0t}\,\hat{z}\,e^{i(\vec{k}_t\cdot\vec{r} - \omega t)}$$

$$\begin{cases} B_{0t} = B_{0i}\,t_{\parallel} \\ k_t = \frac{2\pi}{\lambda} \end{cases}$$

$$\vec{k}_t\cdot\vec{r} = k_t \sin\theta_t\,\hat{x} + k_t\cos\theta_t\,y$$

see Fig 3.10:



For $\theta_i > \theta_c$ and using the expressions for $\sin\theta_t$ & $\cos\theta_t$ derived in class $\left\{\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i,\ \cos\theta_t = i\left(\left[\frac{n_i}{n_t}\sin\theta_i\right]^2 - 1\right)^{1/2}\right\}$.

$$\vec{B}_t = B_{0t}\,\hat{z}\,e^{-\alpha y}\,e^{i(kx - \omega t)}$$

$$\alpha \equiv k_t\left(\left[\frac{n_i}{n_t}\sin\theta_i\right]^2 - 1\right)^{1/2} \qquad k = k_t\left(\frac{n_i}{n_t}\sin\theta_i\right)$$ ✓

(11) (a) Beer's Law: $I = I_0 e^{-\beta z}$ (5)

$$\frac{I}{I_0} = e^{-\beta z} = \frac{1}{2} \quad \text{for } z = 0.10 \text{ cm}$$

$$\ln e^{-\beta z} = \ln \frac{1}{2}$$

$$-\beta z = -\ln 2$$

$$\beta = \frac{\ln 2}{z} = \frac{\ln 2}{0.10 \text{ cm}} = 6.93 \text{ cm}^{-1}$$

$\vec{E} = \vec{E}_0 e^{i(\tilde{k} z - \omega t)}$ and, from lecture,

$$\tilde{k} = k + i \beta/2 = \frac{\omega n_r}{c} + \frac{i \omega n_i}{c}$$

$$n_i = \frac{c}{\omega} \frac{\beta}{2} = \frac{\lambda_0}{2\pi} \frac{\beta}{2} \quad \text{where } \lambda_0 = \text{vacuum wavelength}$$

$$= \frac{(532 \cdot 10^{-7} \text{ cm})}{2\pi} \frac{6.93 \text{ cm}^{-1}}{2}$$

$$= 2.9 \cdot 10^{-5} \checkmark$$

(b) For $\lambda_0 = 575$ nm, f_0 is smaller than for part (a) so we are further from resonance. n_r and n_i should both get smaller.