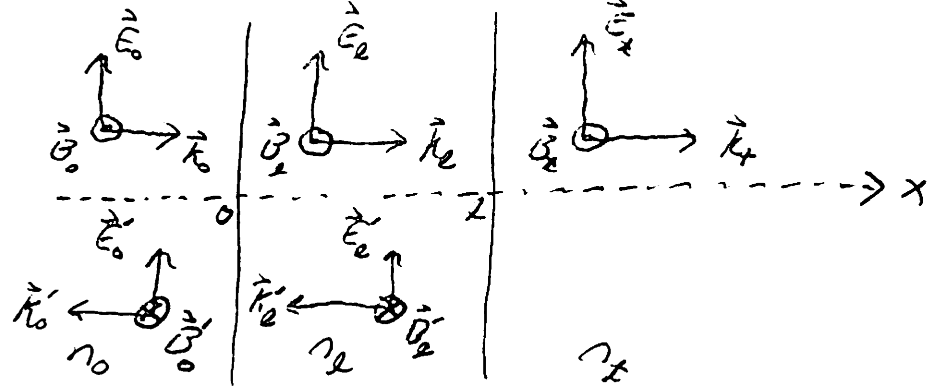
Multilayer Films – a field approach (5.11)



- Assumed direction of fields (take this as a sign convention)
- We need explicitly forward & backward waves since these are linearly independent solutions
- single frequency
- restrict to normal incidence for simplicity

We're looking for a time-independent solution (as usual). Convenient to treat $t=0$, but for $e^{i(kx\pm\omega t)}$, how do we tell the direction?

$\phi = kx \pm \omega t$ is just a convention. We don't have to use it.

$\phi = \pm kx - \omega t$ will work. (Calculate v_p as a check!)

Here: $e^{ikx} \rightarrow$ forward

$e^{-ikx} \rightarrow$ backward

Choose:

$\overset{\text{Given}}{\vec{E}_0 e^{ik_0 x}}$ $\quad \vec{E}_\ell e^{ik_\ell x}$ $\quad \vec{E}_t e^{ik_t(x-\ell)}$ (transmission)

$\vec{E}_0' e^{-ik_0 x}$ (r) $\quad \vec{E}_\ell' e^{-ik_\ell x}$

Any required phases will go into the $\vec{E}$.

Boundary conditions

$\vec{E}$ & $\vec{B}$ are pure tangential and must be continuous:

$\boxed{x=0}$ $E_0 + E_0' = E_\ell + E_\ell'$ (a)

$B_0 - B_0' = B_\ell - B_\ell'$ but $B = \frac{E}{v} = \frac{nE}{c}$

$n_0(E_0 - E_0') = n_\ell(E_\ell - E_\ell')$ (b)

$\boxed{x=\ell}$ $E_\ell e^{ik_\ell \ell} + E_\ell' e^{-ik_\ell \ell} = E_t$ (c)

$n_\ell(E_\ell e^{ik_\ell \ell} - E_\ell' e^{-ik_\ell \ell}) = n_t E_t$ (d)

Only k_ℓ was needed, so let's relabel it: k.
(non-normal incidence not harder, but messy.)
We need E_ℓ and E_ℓ' so we can eliminate them.

(c) + $\frac{(d)}{n_\ell}$ $\Rightarrow$ $2E_\ell e^{ik\ell} = E_t\left(1 + \frac{n_t}{n_\ell}\right)$

$$E_\ell = \frac{1}{2}\left(1 + \frac{n_t}{n_\ell}\right) E_t e^{-ik\ell}$$

(c) $-$ $\frac{(d)}{n_\ell}$ $\Rightarrow$ $E_\ell' = \frac{1}{2}\left(1 - \frac{n_t}{n_\ell}\right) E_t e^{ik\ell}$

Look at (a) & (b): we need $E_\ell \pm E_\ell'$

$$E_\ell + E_\ell' = \frac{1}{2} E_t \left[e^{ik\ell} + e^{-ik\ell} - \frac{n_t}{n_\ell}\left(e^{ik\ell} - e^{-ik\ell} \right) \right]$$

$$= E_t \left(\cos k\ell - i \frac{n_t}{n_\ell} \sin k\ell \right)$$

$$E_\ell - E_\ell' = E_t \left(-i \sin k_\ell + \frac{n_t}{n_\ell} \cos k\ell \right)$$

$$\frac{ⓐ}{E_0} \Rightarrow 1 + \frac{E_0'}{E_0} = \left(\cos k\ell - i \frac{n_t}{n_\ell} \sin k_\ell \right) \frac{E_t}{E_0}$$

$$\frac{ⓑ}{E_0} \Rightarrow n_0 - n_0 \frac{E_0'}{E_0} = \left(-i n_\ell \sin k\ell + n_t \cos k\ell \right) \frac{E_t}{E_0}$$

That's it! Define: $r \equiv \frac{E_0'}{E_0}$ $\quad t \equiv \frac{E_t}{E_0}$

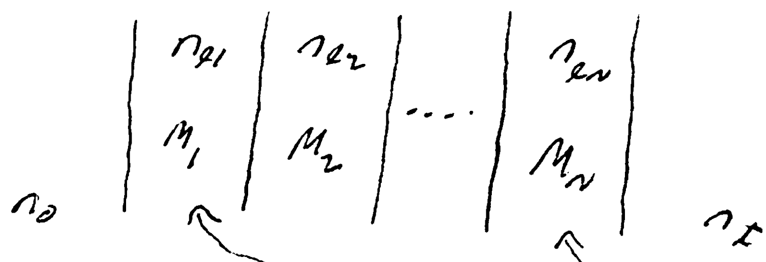
As you'll see, a matrix formulation is useful:

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} r = M \begin{pmatrix} 1 \\ n_t \end{pmatrix} t \quad \checkmark$$

$$M = \begin{pmatrix} \cos k\ell & -\frac{i}{n_\ell} \sin k\ell \\ -i n_\ell \sin k\ell & \cos k\ell \end{pmatrix}$$

Only depends on layer quantities: $n_\ell \quad \ell \quad k = k_\ell$

If there are multiple layers, it turns out their <u>transfer matrices</u> M_ℓ just multiply (see p. 254, footnote 18)



$$\begin{pmatrix}1\\ n_0\end{pmatrix} + \begin{pmatrix}1\\ -n_0\end{pmatrix} r = \underbrace{M_1 M_2 \cdots M_N}_{\equiv \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{pmatrix}} \begin{pmatrix}1\\ n_t\end{pmatrix} t$$

$$1 + r = (m_{11} + n_t m_{12})\, t$$

$$n_0 - n_0 r = (m_{21} + n_t m_{22})\, t$$

$$1 + r = \frac{(m_{11} + n_t m_{12})(n_0 - n_0 r)}{m_{21} + n_t m_{22}}$$

$$r(m_{21} + n_t m_{22} + n_0 m_{11} + n_0 n_t m_{12}) = n_0 m_{11} + n_0 n_t m_{12} - m_{21} - n_t m_{22}$$

$$r = \frac{n_0 m_{11} + n_0 n_t m_{12} - m_{21} - n_t m_{22}}{n_0 m_{11} + n_0 n_t m_{12} + m_{21} + n_t m_{22}} \quad \& \quad R = |r|^2$$

$$t = \frac{2 n_0}{n_0 m_{11} + n_0 n_t m_{12} + m_{21} + n_t m_{22}} \qquad T = |t|^2$$

Back to the AR-coating

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} \cos k\ell & -\frac{i}{n_\ell}\sin k\ell \\ -i\, n_\ell \sin k\ell & \cos k\ell \end{pmatrix}$$

Typically, $n_0 = 1$.

Previously, we found $\ell = \lambda/4 \quad (\lambda = \lambda_0/n_\ell)$

Then: $k\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \begin{pmatrix} 0 & -i/n_\ell \\ -i n_\ell & 0 \end{pmatrix}$

$$r = \frac{n_t\left(-i/n_\ell\right) + i n_\ell}{n_t\left(-i/n_\ell\right) - i n_\ell} = \frac{n_t - n_\ell^2}{n_t + n_\ell^2}$$

$r \neq 0$ in general - extra reflections.

$r = 0$ for $n_\ell = \sqrt{n_t}$ ✓

For generic glass $n_t \approx 1.5$: $n_\ell = 1.22$

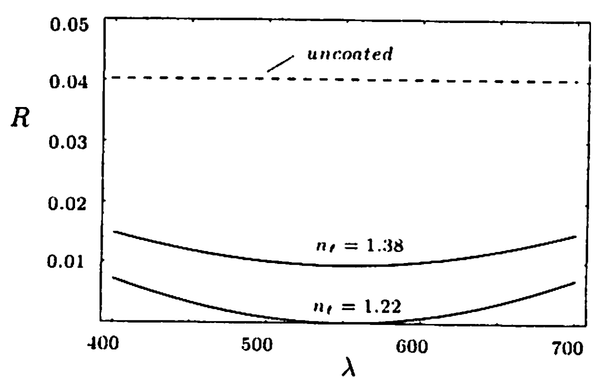


Figure 5.33 Antireflection coating consisting of a single quarter-wavelength layer applied to glass of index 1.5. The ideal index of $\sqrt{1.5}$ gives zero reflection at the design wavelength of 550 nm. Magnesium fluoride with index 1.38 gives a reflection of only about 1% at 550 nm. The uncoated reflectance is about 4%.

It turns out you can do better by adding more layers. Note, in the strategies described below, each layer has $k\ell = \frac{\pi}{2}$ or $\ell = \lambda/4 = \frac{1}{4}\frac{\lambda_0}{n}$

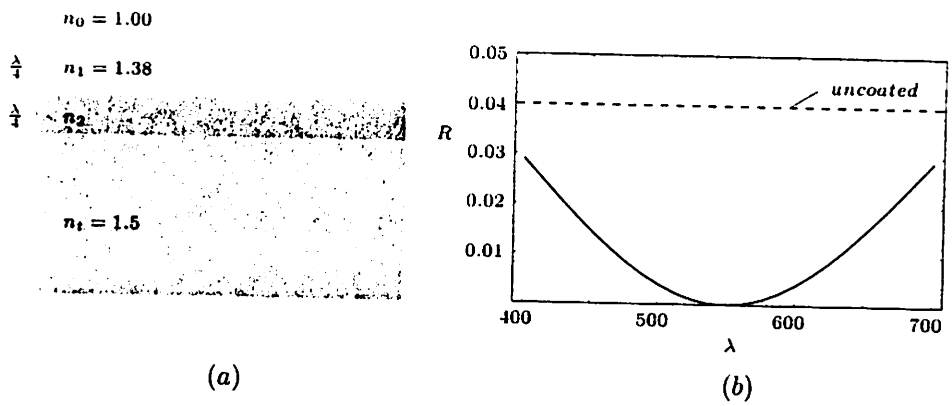


Figure 5.34 Quarter-quarter coating on a substrate with index 1.50. The top layer is MgF_2. (b) Reflectance when the n_2 layer has the ideal index of 1.69.

$$M = M_1 M_2 = \begin{pmatrix} 0 & -i/n_1 \\ -in_1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i/n_2 \\ -in_2 & 0 \end{pmatrix} = \begin{pmatrix} -n_2/n_1 & 0 \\ 0 & -n_1/n_2 \end{pmatrix}$$

$$r = \frac{n_2^2 - n_t n_1^2}{n_2^2 + n_t n_1^2} = 0 \quad \text{when} \quad \frac{n_2}{n_1} = \sqrt{n_t}$$

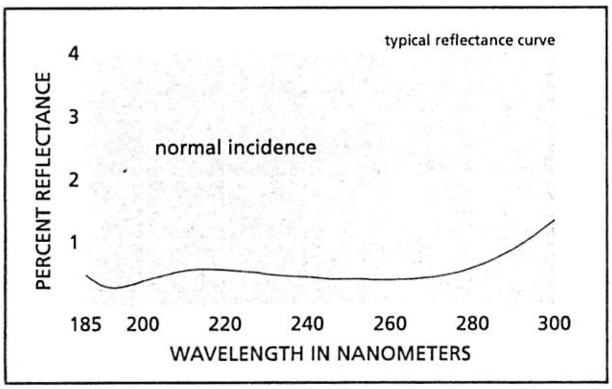
You can't choose n_t typically, but this allows you to use material combinations to satisfy the constraint. This is more likely than $n_1 = \sqrt{n_t}$

In practice, $\approx 0.2\%$ reflectivity is achieved for a single λ_0. Less than 1% over a broad spectrum can also be done, typically using many layers.

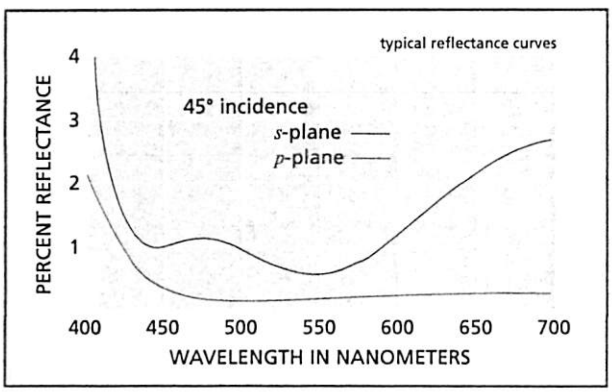
BBAR-Series Coatings

CVI Melles Griot offers six overlapping broad band antireflection (BBAR) coating designs covering the entire range from 193 nm to 1600 nm. This includes very broad coverage of the entire Ti:Sapphire region. The BBDS coatings are unique in the photonics industry by providing both a low average reflection of ≤0.5% over a very broad range and also providing the highest damage threshold for pulsed and continuous wave laser sources (10J/cm^2, 20ns, 20Hz at 1064nm and 1MW/cm@, CW at 1064 respectively). Typical performance curves are shown in the graphs for each of the standard range offerings. If your application cannot be covered by a standard design, CVI Melles Griot can provide a special broad band antireflection coating for your application.

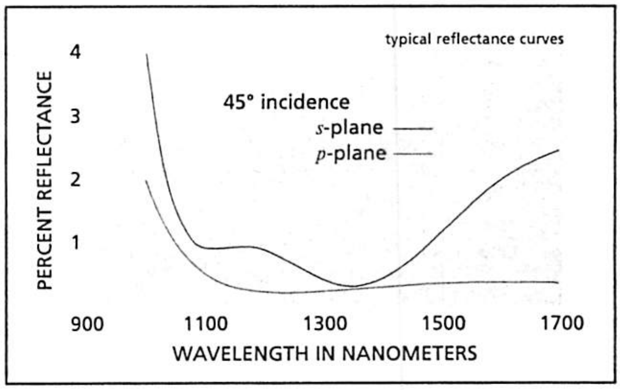
CVI Melles Griot also provides three mid infrared and far infrared broad band antireflection coatings from 2.0 μm to 12.0 μm. These coatings are available on a wide range of materials including Si, Ge, ZnS, ZnSe, or CaF2. Our standard coatings cover 2 to 2.5 μm, 3 to 5 μm and the 8 to 12 μm region. Custom coatings are also available for mid and far infrared applications.



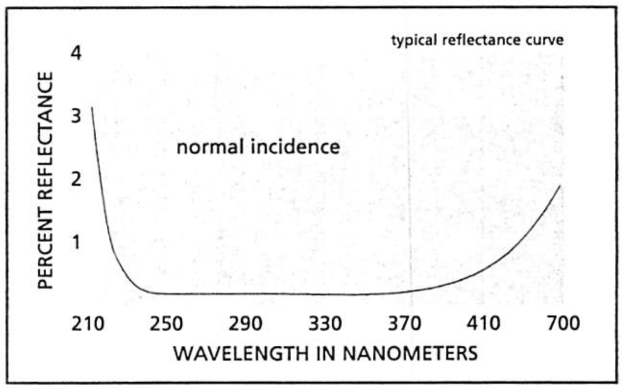
BBAR 193-248 coating for the UV region (0° incidence)



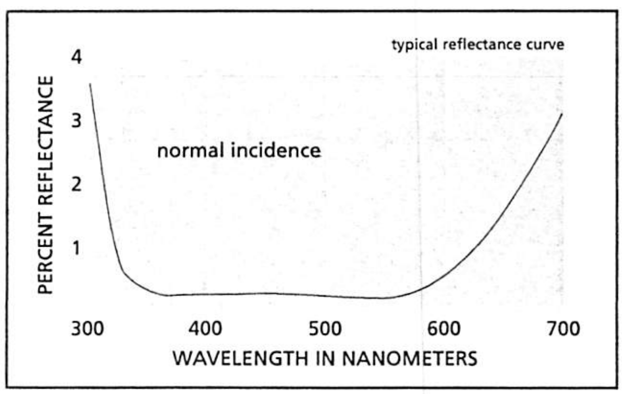
BBAR/45 425-675 coating for the visible region (45° incidence)



BBAR/45 1050-1600 coating for the NIR region (45° incidence)



BBAR 248-355 coating for the UV region (0° incidence)



BBAR 355-532 coating for the UV region (0° incidence)

HR Coatings

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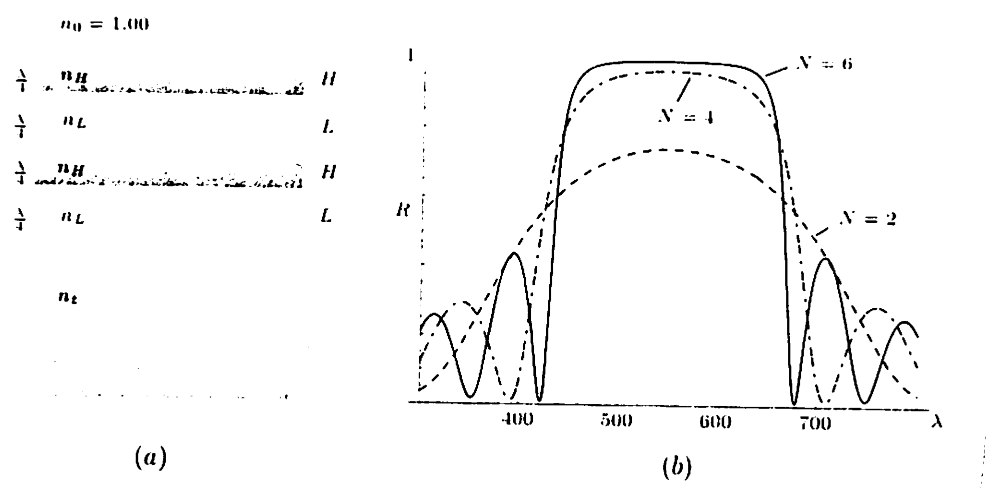

Figure 5.35 (a) A two-pair high-reflectance coating. (b) Reflectance vs. wavelength for 2, 4 and 6 pair coatings. See Example 5.12.

One HL pair:

$$M_{HL} = M_H M_L = \begin{pmatrix} -n_L/n_H & 0 \\ 0 & -n_H/n_L \end{pmatrix}$$

see notes p 94

N pairs:

$$M = M_{HL}^N = \begin{pmatrix} \left[-\frac{n_L}{n_H}\right]^N & 0 \\ 0 & \left[-\frac{n_H}{n_L}\right]^N \end{pmatrix}$$

$$r = \frac{\left[-\frac{n_L}{n_H}\right]^N - n_t\left[-\frac{n_H}{n_L}\right]^N}{\left[-\frac{n_L}{n_H}\right]^N + n_t\left[-\frac{n_H}{n_L}\right]^N}$$

$$R = |r|^2 = \frac{\left(\frac{n_L}{n_H}\right)^{2N} - 2\left(\frac{n_L}{n_H}\frac{n_H}{n_L}\right)^N n_t + n_t^2\left(\frac{n_H}{n_L}\right)^{2N}}{'' \quad + \quad '' \quad + \quad ''}$$

$$R = \left|\frac{1 - 2n_t + n_t^2\left(\frac{n_H}{n_L}\right)^{4N}}{1 + 2n_t + n_t^2\left(\frac{n_H}{n_L}\right)^{4N}}\right| = \left|\frac{n_t\left(\frac{n_H}{n_L}\right)^{2N} - 1}{n_t\left(\frac{n_H}{n_L}\right)^{2N} + 1}\right|^2 \to 1$$

$N \to \infty$ for $\frac{n_H}{n_L} > 1$ ✓