Homework Set #4 **Due: 2-10-12**

1) A TEM_{00} mode is incident upon a positive lens. Let R_{in} be the radius of curvature at the lens. Show that if $0 < R_{in} < f$, the output beam will not converge. In analogy to the equivalent ray-optics problem, you could say that the waist of the output beam was "virtual". Find the location of the virtual waist.

2) 4.8 (This is a useful rule of thumb. Note that for a uniform spatial profile, $P = AI$).

3) 4.9

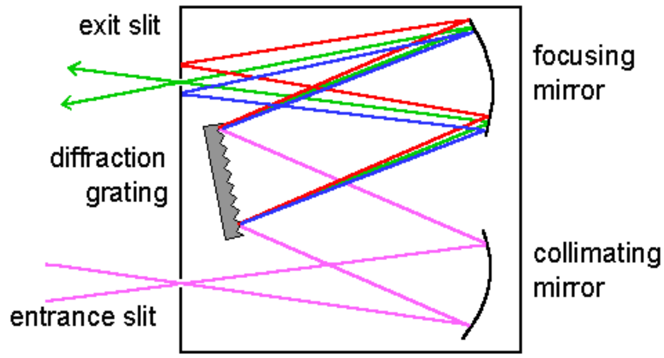
4) 4.12 (Compare to the solution for the "spot on the moon" homework assignment. Both approaches take diffraction into account, but with different assumptions.)

5) Given: A TEM_{00} mode with wavelength λ and waist w_o (located at z=0), a lens at $z = \ell$ and a target at $z = \ell + L$. **(a)** Find the focal length of the lens so that a waist is at the target. **(b)** You can think of this as an imaging problem: the goal is to image the waist at z=0 onto the target. If the ray optics limit held, then $1/f = 1/o + 1/i = 1/\ell + 1/L \rightarrow f = \ell L/(\ell + L)$. Compare this result to your solution in the appropriate limit.

This problem describes a common scenario in many laser labs and is often referred to as "beam relaying". High power lasers often have modes that degrade due to diffraction as they propagate but, so long as the mode is of acceptable quality somewhere, that mode can be relayed to a target with appropriate magnification.

6) ***The Monochrometer.***

A monochrometer is used to analyze spectra and a classic example is shown in the figure. The "slits" are narrow openings into and out of the device. Assume that light is focused onto the entrance slit, both mirrors have focal length f, and the distance between each slit and its nearest mirror is f. Applying a ray optics analysis, light from the entrance slit is collimated by the first mirror, dispersed by the grating, and focused onto the exit slit. As you saw from the last problem in the last homework assignment, each color will be focused to a different location and only a small bandwidth from the entire spectrum will pass through the exit slit. The grating angle determines which wavelength passes through the slit. A simple power meter placed after the exit slit can record the signal as the grating angle is varied, thus measuring the spectrum of the light. If the exit slit is removed and a spatially resolved detector is put in its place (e.g. a CCD array or a piece of film), a large spectral range can be measured at one time. When used this way, the apparatus is called a spectrometer.



Schematic of a Czerny-Turner monochrometer. *Image credit:* www.files.chem.vt.edu/chem-ed/optics/selector/spectrom.html

For the system above, assume the slit widths are 30 μm, f = 200 mm, the grating has 500 lines/mm and is operating in first order, the incident angle is 45^o, and the center wavelength of the incident light (from a laser) is 500 nm. The grating itself is in the shape of a square 2" on a side.

(a) Often, experimenters will directly illuminate the entrance slit with a laser beam rather than focus onto it. Most of the laser energy won't pass through the slit, but the beam is so bright there is plenty of signal anyway. However, this can greatly reduce the monochrometer's resolution. The grating is only of use if a large number of the diffraction grooves are actually illuminated. If $\Delta\lambda$ is the wavelength resolution, we have $\Delta\lambda/\lambda = mN$ where λ is the center wavelength, m the diffraction order, and N is the number of grooves illuminated. Estimate what the resolution is for this case by applying single slit diffraction theory *to the entrance slit* to see what effect it has.

(b) The input slit is removable so that different width slits can be used. For the situation in part (a), why might it be stupid to remove the slit without blocking the laser first?

(c) What is the resolution if the laser is properly focused onto the slit so as to fill the grating? By what factor is the resolution improved from the situation in part (a)? (Note that if you fill the grating horizontally, the beam will be too large vertically and some of the light will miss the grating.) As you saw in class, the "f number" is a measure of beam divergence. Manufacturers specify an f-number for their monochrometers to get full resolution.

(d) Ideally, the bandpass through the slit is matched to the grating resolution. What is the bandpass?