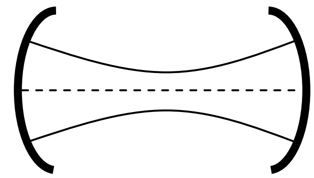
Homework Set #5 **Due: 2-20-12**

(1) Text 5.8.

Always include a quick sketch of what the stable mode looks like for problems like this. For example, for a symmetric cavity close to confocal, you could simply draw something like:



(2) Text 5.11.

(3) Text 5.14.

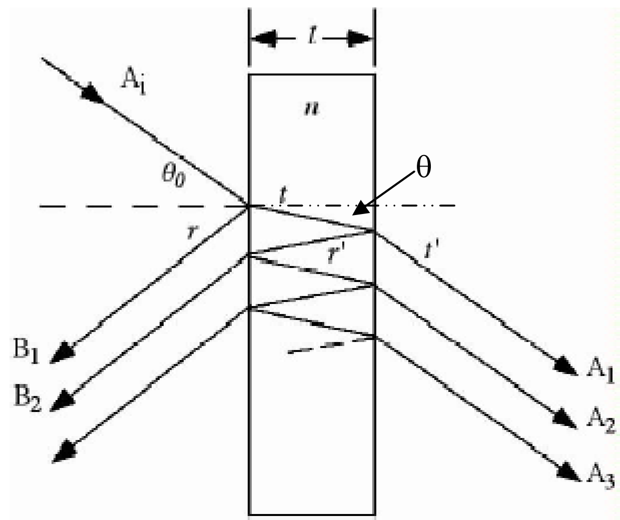
Assume a symmetric confocal cavity.
As discussed in lecture, Siegman has a nice treatment of sensitivity to misalignment.

(4) Text 5.15.

My answer differs from the text by a factor of 4 and the denominator should be w^3, not w^2.

5) The Etalon.

In this problem you'll derive the basic properties of a Fabry-Perot etalon. There are many different types of etalon, but this problem captures the essentials. Etalons are most commonly used for spectral measurement, but they are also used for spectral shaping in laser systems, particularly narrow line lasers, but also short pulse lasers. A laser cavity is itself an etalon, so this problem will help aid our understanding of laser operation. The main goal here, however, is simply to get a physical feel for what an etalon is and how it acts as a spectral filter by using wave interference. This is a classic problem and there are plenty of references (including me!). Note, there is an alternate solution that simply involves matching boundary conditions between fields inside and outside the etalon. Feel free to take that approach if you like.



In the figure, light of wavelength λ is incident on a rectangular medium of index n with flat, parallel sides and dimensions as shown. A medium in this shape is often called a "window" or a "flat". There is no absorption, but both sides of the flat are coated so that the electric field reflectivity of each face is r and r'. Thus, if A is the complex electric field magnitude of an incident plane wave, $A_{reflected} = r * A_{incident}$, for the left face. Likewise, the electric field transmissions are t and t'. For this problem, assume that r = r', t = t', there is no absorption (so $r^2+t^2=1$) and there is no phase shift due to reflection at an interface. The last is clearly wrong, but these assumptions do not throw away any interesting physics and simplify the math.

(i) We can write $A_1 = t^2 A_i$. Likewise A_2 can be written as $A_2 = C e^{i\delta} A_i$. The factor $e^{i\delta}$ accounts for the phase shift between the outgoing fields A_1 and A_2 because of the extra path length traveled for A_2. Find C and δ in terms for t,r,n,l,θ,λ. Explain why (or if) I am allowed to neglect the phase shift in my expression for A_1.

(ii) Write the total transmitted field, A_t, as an infinite series of terms: $A_t = A_1 + A_2 + \dots$ This series converges to a finite result, so find a simple analytic expression for A_t.

(iii) Show that the fractional transmitted power, $T = |A_t|^2 / |A_i|^2$ is given by:

$$T = \frac{1}{1+F\sin^2(\delta/2)} \text{ with } F = \frac{4R}{(1-R)^2} \text{ and } R = r^2 .$$

R is the reflection coefficient for the intensity.

(iv) The transmission is periodic in frequency. In other words, light with a frequency of:

$$\nu_n = m\Delta\nu$$

will be completely transmitted with no reflection losses, even if R = 99.999%! Δν is called the free spectral range (FSR). Find the FSR. Sketch T versus frequency.

(v) Sketch the transmission function over several free spectral ranges for two values of F: F_1 and F_2 where $F_1 < F_2$. Convince yourself that F determines the "bandpass" of the etalon.