Homework Set #7 **Due: 3-7-10**

1) text 7.3

2) text 7.5

3) text 7.6

4) text 7.14

5) Determine the behavior of a 4-level laser starting from when it is first pumped until it reaches steady state.

The gain medium has these properties: τ_2 = 30 μs, σ_{12} = 3.0 x 10^{-19} cm^2, ΔE_{12} = 1.24 eV, ℓ = 10 cm and D = 4.0 mm (upper state lifetime, transition cross-section, transition energy, medium length and medium diameter respectively).

The cavity is described by: L_{opt} = 100 cm, R_2 = 95% and L_i = 3.0% (optical path length, output coupler and internal loss respectively).

Assume a stable cavity and constant pumping at 5 times threshold. For simplicity, also assume that the laser mode has a uniform cross-section with the same diameter as the gain medium.

(a) Present your results using two well-labeled graphs. The first should plot the population inversion and number of cavity photons from start until steady state is reached. The second should plot the same quantities, but as an expanded view of the first relaxation oscillation. Include a terse description of your numerical integration technique, just as you did in the previous homework assignment.

(b) I can think of three potentially important time scales in the problem: the round-trip travel time, the cavity lifetime and the upper-state lifetime. Are there any features in the above graphs that can be associated, even if roughly, with these time scales? Make sure to explain why a given feature is present.

(c) Compare your numerical results for the steady state inversion and photon number to the predicted values. *These must agree within 1% for full credit.*

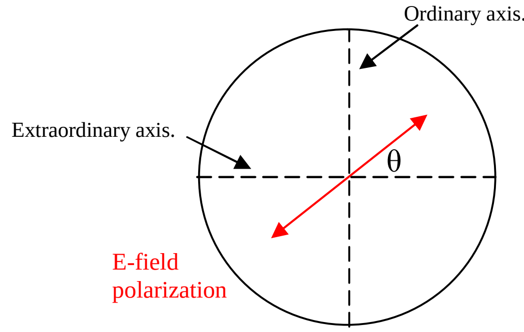
(d) What is the output power of this laser? If the gain medium is pumped with perfect coupling and absorption efficiency using an 800 nm diode-bar laser, what pump power is required?

(e) Keeping the same numerical value for the pump rate as used for the above, what would be the output power of this laser if its output coupling had been chosen correctly?

Suggestions: As discussed in class and described in the text, you should expect spikes in the output occurring over a long time scale. This can be a challenging combination for numerical analysis. Select your time step accordingly. (Question: What is the correct time scale to use when picking your initial time step? Answer: Look at the equations of motion!) Assume that spontaneous emission guarantees that there is at least one photon in the correct mode - don’t allow the photon number to drop below one. Finally, this assignment differs from the previous one in that you must integrate two coupled equations at the same time: one for ϕ and one for N. The procedure is similar, however.

(6) The Waveplate

Waveplates are important optical components that allow modification of the polarization of light. "Half-wave" plates rotate the polarization direction and "quarter-wave" plates can transform linear polarization to elliptical polarization or circular polarization and back.



In the following discussion, make sure to distinguish when I am referring to the direction of the light versus the polarization direction.

The key idea to understanding waveplates is that they are made of anisotropic media. <u>Anisotropic media will in general have an index of refraction that depends on the polarization direction.</u> This is because it is easier to shake the electrons in the medium in one direction than in another. However, the index of refraction cannot be written as a simple function of polarization direction for these media because <u>a given direction of propagation is now associated with two different indices of refraction</u>. (The reason for this will not be described here. Although this won't help much, I'll note that it is related to the degrees-of-freedom argument for the existence of three moments of inertia in an arbitrary rotating body.) Depending on the polarization direction, the light can travel with a phase velocity given by one, the other, or both indices of refraction. In this latter case, it effectively splits into two beams. For example, if light enters an anisotropic medium at non-normal incidence, it will generally split into two beams each refracting at a different angle corresponding to the two indices of refraction. This is called "double refraction".

Thus, for a given crystal and direction of the light, two polarization directions exist such that the light will propagate with an unchanging polarization and characterized by a single index of refraction. These are referred to as the extraordinary and ordinary modes. Suppose we have a thin plate made of an anisotropic crystal (for waveplates, quartz is usually employed) oriented so that, for linearly polarized light at normal incidence, vertical polarization is ordinary ($n = n_o$) and horizontal polarization is extraordinary ($n = n_e$) with $n_o \neq n_e$ (for quartz, $n_e < n_o$). If the polarization is initially polarized pure ordinary, it will remain linearly polarized and travel with phase velocity c/n_o; likewise for pure extraordinary, but with phase velocity c/n_e. If the polarization is not initially parallel to either axis, then the polarization of the light will not be preserved as the light propagates through the crystal, because it now consists of two components traveling at different speeds causing them to fall out of phase.

To understand this, we can always break the polarization vector of the input light into components parallel to either axis. Thus, we take the electric field at the input face of the crystal to be:

$$\vec{E} = E_o(\cos\theta\,\hat{e} + \sin\theta\,\hat{o})$$

where $\hat{e}$ and $\hat{o}$ are unit vectors pointing along the extraordinary and ordinary axes (to the right and up, respectively, in the figure), and θ is the angle the polarization vector makes with the extraordinary axis.

[The following problems can be solved as is or, if you prefer, feel free to switch to a complex representation of the field. That almost always simplifies working with light.]

(a) *Half-wave plate.* Suppose we have monochromatic light, of wavelength λ, with input polarization as given above. At the output of the crystal, the ordinary and extraordinary components will have a phase difference between them because they traveled through different optical pathlengths. Suppose the thickness of the crystal is selected such that, after propagating through it, this <u>phase difference</u> is $(N + ½) * 2\pi$ where N is a positive integer. For ease of construction and for strength the plates are usually ~1 mm thick, so N is large.

   (i) Find an expression for the length of the crystal given n_o, n_e, and the information above.

   (ii) Show that the polarization at the output end of the crystal is still linear, but that the polarization vector is now rotated. Determine how the output polarization angle is related to θ.

   (iii) How does the output polarization depend on N?

(b) Qualitatively describe what will happen if a slightly different wavelength, $\lambda' \neq \lambda$, is used. How does your answer depend on N?

(c) *Quarter-wave plate.* Now let the phase shift be $(N+¼) * 2\pi$. Show that the effect of the crystal is to produce elliptical polarization and that for $\theta = 0^o, 45^o, 90^o, 135^o$ the resultant polarization is linear, circular, linear, circular (with opposite handedness) respectively.

<u>Discussion</u>.

Waveplates are made from flats that have sides that are flat and parallel to high precision. This helps avoid unintentional phase shifts. Unfortunately, this means waveplates will also act as Fabry-Perot etalons (with R $\approx$ 5%) and thus have a wavelength dependent transmission. To avoid this, as well as to avoid loss of energy and stray beams, waveplates are always anti-reflection (AR) coated on both sides. Note that a beam sent through a waveplate and telescope, for example, will lose ~27% of its power if all the surfaces are uncoated.

Another difficulty is that the index of refraction seen by the light will change in general if the light is not normally incident upon the waveplate. The ordinary and extradinary polarization directions and their associated indices of refraction depend on input angle. If normal incidence is not used, a half-wave plate will produce elliptically polarized light: the larger the deviation, the worse the effect. If the beam is diverging or converging when it travels through the waveplate, then there is *no well-defined angle of incidence*; a range of angles is present. The output polarization will now vary spatially. For example, for a half-wave plate, the center of the beam would be linearly polarized, but off-center light would be slightly elliptically polarized. Waveplates require careful alignment. In practice, they always degrade the polarization somewhat.

For short pulse work, we must also take into account the consequence of a range of wavelengths being present, as you can see from your answer to part (b).

All of the above effects are minimized if N = 0! Such a waveplate is called a “zero-order waveplate”. They generally aren’t used for monochromatic light, but they are mandatory for short pulse work. The question is, how do you make one? The most direct approach, using a very thin plate, is impractical. You should convince yourself that the flat would have to be a few 10’s of wavelengths long. (This can be done. The optic will be expensive and fragile, however.)

One clever way to do this is to sandwich two flats together, the extraordinary axis of one aligned to the ordinary axis of the other. If the two plates were of identical thickness, there would be no polarization effect at all. By making one of the plates slightly thicker, an effective zero-order half- or quarter-waveplate is obtained. This is usually what is done.