A loose end...

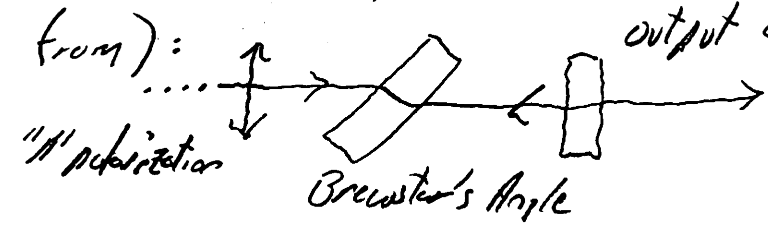
- We've seen how starting from a (possibly sharp) resonance you can get broadening (homogeneous, inhomogeneous) via spon. emission, collisions, Doppler, N.R. transitions, etc.
- Dynamic effects determine the laser output: gain narrowing, frequency pulling, saturation, etc.
- And, the cavity plays a role: longitudinal & transverse modes.

Finally, you can introduce components to control this: prisms, gratings, etalons, and non-linear elements (Kerr media).

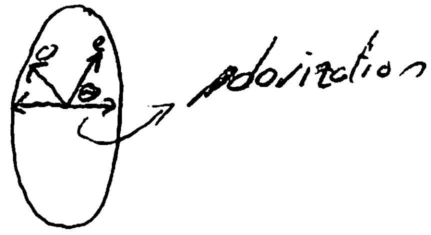
There's always more. Now that you understand waveplates we can cover another element for spectral narrowing and tuning:

The Birefringent Filter (BRF) or Lyot Filter

Start with a quartz crystal plate (same material waveplates are made from):



Looking along the laser axis:



The light has e & o components, so the quartz acts like a waveplate.

F2

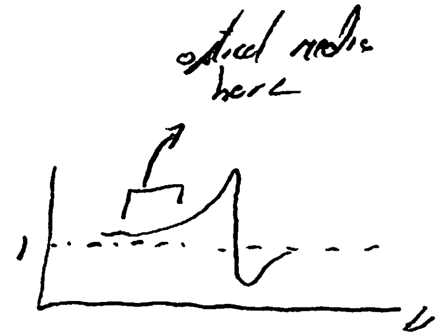
Let the phase shift be $\Delta\phi = N 2\pi$ at $\lambda = \lambda_0$.

Nothing will happen!

Note, if the crystal is thick ($\sim 1\,mm$), N is large and getting $\Delta\phi = N 2\pi$ is not hard. You don't need an exact thickness. Instead, adjust n_e by choosing θ.

$$\Delta\phi = (k_e - k_o)L = (n_e - n_o)\, 2\pi \frac{L}{\lambda}$$

We want: $(n_e - n_o)\frac{L}{\lambda} = \text{integer} = N$



What about $\lambda = \lambda_0 + \Delta\lambda$?

Well, n_o & n_e are functions of λ:

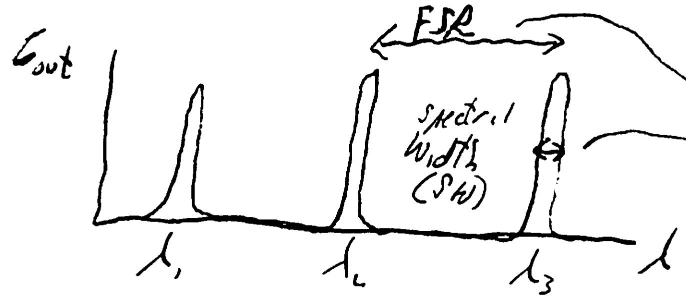
$$\Delta\phi = (N + \varepsilon)\, 2\pi$$

and the light becomes elliptically polarized.
It acquires an "s" component with some phase shift.

If the laser has higher gain for "p" than "s" (and it will, because of the BRF itself, if nothing else) the mixed polarization ($\lambda + \Delta\lambda$) will be suppressed compared to pure p-pol λ.

Of course, if $\Delta\lambda$ is big enough, $\Delta\phi = N' 2\pi$.

If the gain medium had ∞ bandwidth:



Determined by thickness, dispersion, and cavity polarization dependence.

The larger L is: the smaller SW is.
the smaller FSR is.

Changing θ tuned the laser output wavelength.

Combining thick and thin plates will give you:
small SW (thick) and large FSR (thin).

Common combination is something like: $L_1 = 5mm$, $L_2 = 2.5mm$, $L_3 = 1.25mm$.

Characteristics:

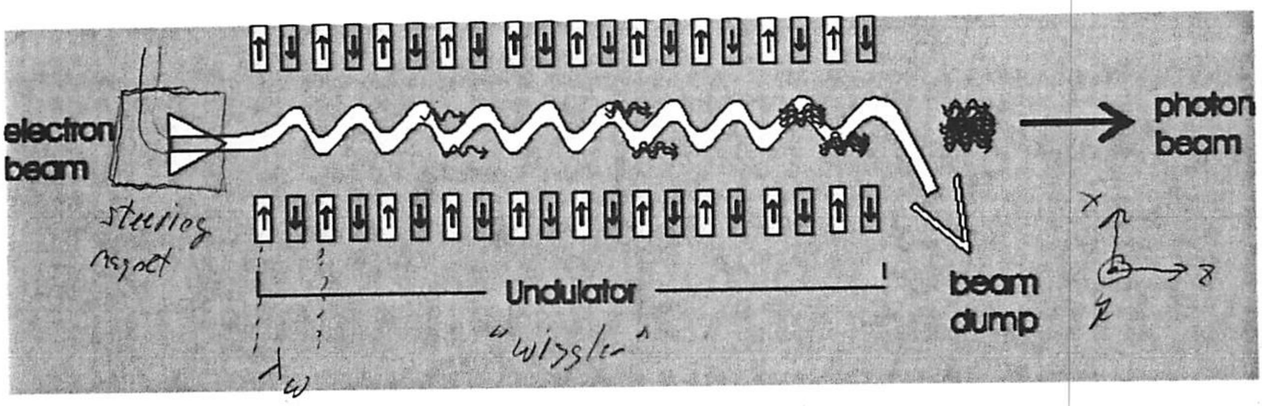
- Small insertion loss (unlike a grating)
- no coatings (unlike etalons)
- can easily be inserted & removed (unlike prisms)
- tuning without steering
- effect is relatively subtle - not for high gain systems.

Generally used in cw systems where you want tuning and don't need super narrow lines.

The Free Electron Laser - FEL

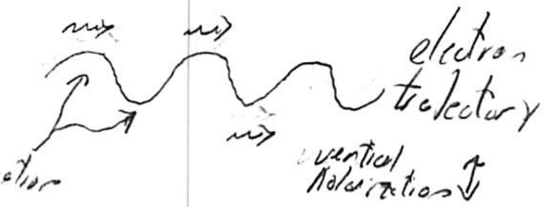
Free electrons (in a spatially varying $\vec{B}$ field) as a gain medium. Electrons must be relativistic.

We'll start with the text: 10.4 (some figures from around the web)
Let's not worry about lasing yet and <u>just consider spon. emission</u>



$B_x = \uparrow \downarrow \uparrow \downarrow$ Electron beam: osc normal to the page.

Spontaneous emission: synchrotron emission



stimulated emission: wiggling electron in the presence of a resonant EM wave

<u>Resonant? What's the frequency?</u>

Electron osc frequency must be $f_e = \dfrac{v_z}{\lambda_w}$ w/ v_z = electron speed <u>averaged over 1 wiggle</u>

$\approx \dfrac{c}{\lambda_w}$

If e^- was osc. in place, this would also be its emission freq. We'll do a rough calculation to deal with this.

B conserves electron kinetic energy. Radiation does not.

Go to e^- reference frame (well, moving at V_z): pure dipole | FS

Freq in rest frame $\equiv f' = \gamma f_e$ (time dilation $\Delta t' = \Delta t/\gamma$)

$$\gamma \equiv \frac{1}{\sqrt{1 - V_z^2/c^2}}$$ (using V_z instead of v - not right, but connects to λ_w)

f' = frequency in electron frame of the emission

Transform back to lab frame:

emission freq $$f = \sqrt{\frac{1+\beta}{1-\beta}}\, f'$$ relativistic Doppler shift $\beta \equiv \frac{V_z}{c}$
source moving $\rightarrow$ shift UP

$$= \frac{1+\beta}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}} f_e$$

$$= \frac{1+\beta}{1-\beta^2} f_e$$ for $\beta \approx 1$

$$f \approx \frac{2}{1 - V_z^2/c^2} f_e$$ $f \gg f_e$

$$\lambda = \frac{\lambda_w}{2}\left[1 - \left(\frac{V_z}{c}\right)^2\right]$$ ✓

For $V_z \approx c$, more convenient to work w/ energy.

$$E = \frac{1}{\sqrt{1 - v^2/c^2}} mc^2 \rightarrow 1 - v^2/c^2 = (mc^2/E)^2$$

We want to use V_z. You can show:

$$1 - V_z^2/c^2 = (1+K^2)\left(\frac{mc^2}{E}\right)^2$$ $$K \equiv \frac{e\langle B^2\rangle^{1/2} \lambda_w}{2\pi mc^2}$$ undulator parameter

$\langle B^2\rangle$ is average down the undulator axis

From all this, you get:

$$f = 2 f_e \overset{\approx \frac{1}{2}}{\left(\frac{1}{1+K^2}\right)}\left(\frac{E}{m_0 c^2}\right)^2 \rightsquigarrow \text{big}$$

So, we can tune the frequency via λ_w, B, E.

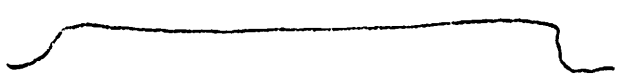
E is easiest ⇨ For $\lambda_w = 10$ cm & $K \approx 1$

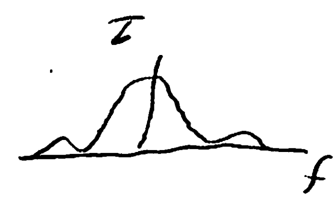
$E = 10^2$ to 10^3 MeV goes from IR to UV.

That's the resonance. What's the line shape?

Electron motion:  for N cycles where $N = \frac{\text{length}}{\lambda_w}$

$\vec{E}$: 

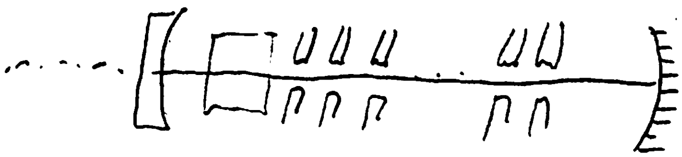
I 

Spectrum of a rectangle envelope is an Airy pattern 

Since each electron is the same, homogeneously broadened.

(Inhomogeneities in in field and differing trajectories give rise to an inhomog. contribution.)

Lasers (original approach)



The first FEL used a pulsed electron beam (accelerators generally run pulsed) so T_{RT} was adjusted to match the pulse period : synch. modelocking.

FEL's like this are expensive because of the accelerator but, they are widely tunable and high power since accelerator physicists are good at making high current, high rep rate e^- beams.

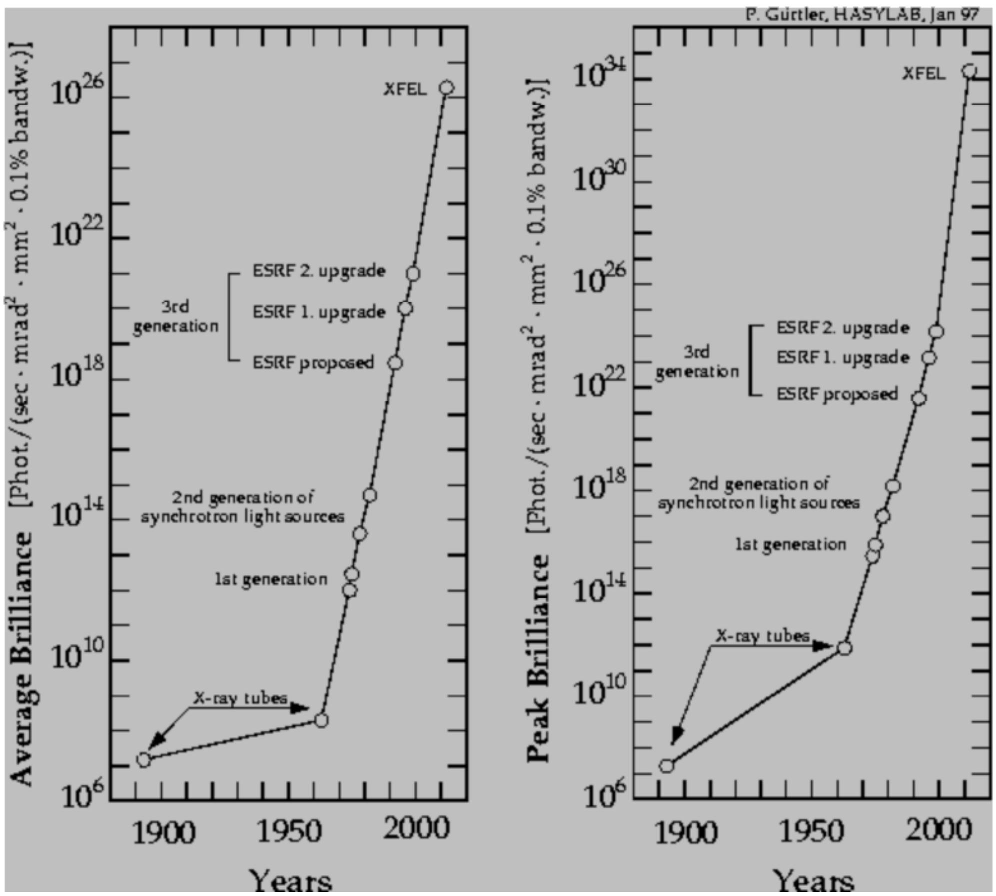
FELs are usually employed to get at difficult parts of the spectrum: Far IR, VUV, and X-ray.

X-ray FEL (XFEL)

"Soft" x-rays begin around 100 eV.
1 – 100 keV photons are x-ray (below 1 nm).
Gamma rays are above that.

From: xdb.lbl.gov/Section2/Sec_2-2.html
"1stgeneration" x-ray sources – accelerators and synchrotrons designed for particle research,
but supporting x-ray generation "parasitically".
2nd generation – systems designed with x-ray generation in mind.
3rd generation – storage rings with undulators.
4th generation – *linear* accelerators with very long undulators, using self-amplified spontaneous emission (SASE).

SASE-FELs use a single pass through the undulator.



From: www.gcpd.de/publication/gcpd00/shiwenlong.pdf.
ESRF = European Synchrotron Radiation Facility.

(Some of the following notes taken from: www.slac.stanford.edu/pubs/beamline/32/1/32-1-pellegrini.pdf,
A non-mathematical tutorial from the LCLS = Linac Coherent Light Source. Linac = linear accelerator.)

So, here's where we are:
(emittance is a measure for the average spread of particle coordinates in position-and-momentum phase space)

Undulator Radiation Characteristics

Some typical characteristics of the undulator radiation from third-generation ring-based light sources, and free-electron laser light sources. The emittance is in nm rad; the pulse length in picoseconds; the average and peak brightness are in photons/sec/mm^2/mrad^2/0.1% bandwidth; the peak power in watts.

	Third Generation	*SASE-Free-Electron Laser*	*Short pulse SASE-Free-Electron Laser*
Wavelength range, nm	1-0.1	1-0.1	1-0.1
Emittance, nm-rad	2	0.03	0.03
Pulse length, ps	15-30	0.06	0.01
Average brightness	10^{20}	10^{22}	10^{21}
Peak brightness	10^{23}	10^{33}	10^{33}
Peak power, W	10^{3}	10^{10}	10^{10}

LCLS Parameters

ENERGY SPREAD, PULSE LENGTH, emittance are rms values. Brightness is in the same unit as the earlier table. The energy spread is the local energy spread within 2π cooperation lengths. A correlated energy chirp of 0.1 percent is also present along the bunch.

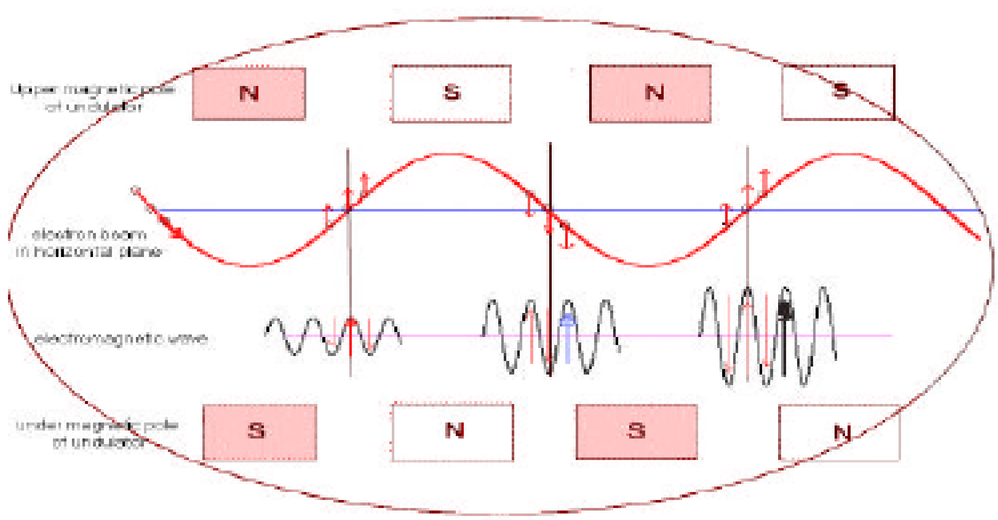
Electron Beam	
Electron energy, GeV	14.3
Emittance, nm rad	0.05
Peak current, kA	3.4
Energy spread, %	0.02
Pulse duration, fs	230
Undulator	
Period, cm	3
Field, T	1.32
K	3.7
Gap, mm	6
Total length, m	100
Radiation	
Wavelength, nm	0.15
FEL parameter, ρ	5×10^{-4}
Field gain length, m	11.7
Bunches/sec	120
Average Brightness	4×10^{22}
Peak brightness	10^{33}
Peak power, GW	10^{10}
Intensity fluctuations, %	8

How does SASE work? An electron pulse is injected into the undulator. Initially you get spontaneous emission, which then follows its electron bunch. (Note in the figure below that the EM field has a smaller wavelength than the wiggler period.) That's the spontaneous emission part of SASE.

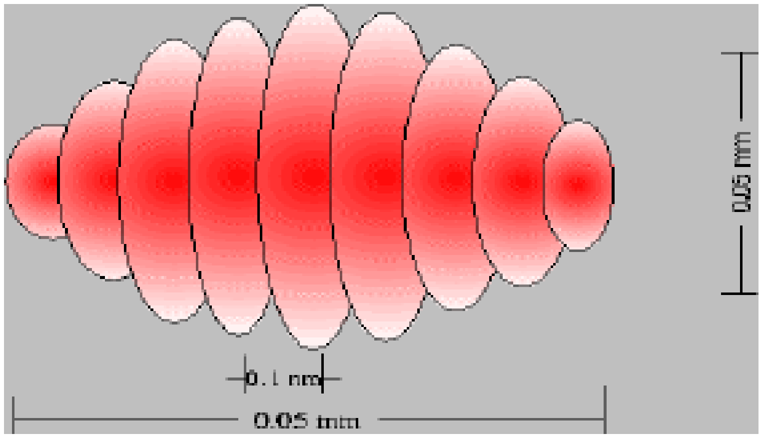
The electric field will act to slow some electrons down and speed others up. Where the electrons lose energy, the field gains it.

(This is stimulated emission. The mechanism whereby the field gains energy via the electron emission .)

For example, the middle arrow in each case below shows an electron moving in the direction of the field (opposite to the force), and so is slowing. The left arrow points to an electron that is accelerating. Thus, the electrons tend to bunch with a wavelength matching that of the light: microbunching.



The electron pulse, after microbunching, looks something like:



So, initially we had an electron pulse incoherently emitting x-rays all across the pulse.
The incoherent sum of N emissions just goes as N.

After microbunching, we have emission primarily from the regularly spaced bunches and they are phased with each other. Now we have a coherent sum going as N^2. This drives the microbunching more strongly, and so forth.

The incoherent electron pulse and the incoherent spontaneous emission “pick” out a coherent response from each other. That’s the “self-amplification” part of SASE. You need the initial electron pulse or emission to be noisy enough that it will contain, among the many modes present, one that can start this process and amplify up. The electron beam must initially be fairly monoenergetic, or dispersion will smear the bunches out. There are many restrictions and it’s hard than this sketch makes it sound.

This process saturates when the electrons are all well-phased. This happens in ~1000 undulator periods.