Quiz 8

(a)

The easiest way to tackle this is using energy conservation,

\[ E_i = E_f \]

\[ KE_i + PE_i = KE_f + PE_f \]

\[ \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f \]

Since the system starts from rest the initial kinetic energy is zero. Also, \( v \) and \( \omega \) are related by \( v = r\omega \). \( I \) here is the moment of inertia of the disk which is \( 1/2MR^2 \). Thus, we have,

\[ 0 + 0 + mg(h_i - h_f) = \frac{1}{2}mv_f^2 + \frac{1}{2} \left( \frac{1}{2}MR^2 \right) \left( \frac{v_f}{R} \right)^2 \]

\[ v_f^2 = \frac{mg(h_i - h_f)}{\frac{1}{2}m + \frac{1}{4}M} \]

\[ v_f = \sqrt{\frac{mg(h_i - h_f)}{\frac{1}{2}m + \frac{1}{4}M}} \]

Plugging in the numbers, we have,

\[ v_f = 5.715 \text{ m/s} \]

(b)

The acceleration of the 5 kg mass can be calculated by using kinematics,

\[ v_f^2 = v_i^2 + 2a(h_f - h_i) \]

\[ 5.715^2 = 0 + 2a(2) \]

\[ a = 8.167 \text{ m/s}^2. \]

(c)

The angular acceleration and linear acceleration are related by \( a = R\alpha \). Thus, the angular acceleration of the disk is

\[ \alpha = \frac{8.167}{0.1} = 81.67 \text{ rad/s}^2. \]