Name (1 pt): ____________________________

Recitation Instructor (1 pt):______________________________

There are four pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

Problem 1.1 (10 pts) The distance to the sun is \(1.5 \times 10^{11}\) m and its diameter is \(1.4 \times 10^9\) m. You produce an image of the sun using a thin lens whose focal length is 20 cm. What is the diameter of the image?

\[
\frac{1}{f} + \frac{1}{i} = \frac{1}{o} \Rightarrow \frac{1}{i} - \frac{1}{f} = \frac{1}{o} \Rightarrow i = \frac{20 \text{ cm}}{1.3 \times 10^{-12}}
\]

Then, \(\text{image diameter} = |\text{magnification}| \cdot \text{object diameter} = 1.3 \times 10^{-12} \times 1.4 \times 10^9 = 1.32 \times 10^{-3}\) m

Problem 1.2 (10 pts): A ray of light is brought perpendicularly to one face of a \(30^\circ - 60^\circ - 90^\circ\) prism as shown in the figure. If the prism is setting in air, what is the minimum index of refraction for the material from which it is made so that the ray of light will undergo total internal reflection at its reflection from the hypotenuse?

From geometry, the incident ray makes an angle \(60^\circ\) with the normal.

Then, \(n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow n_1 \sin \theta_1 = 1 \cdot \sin 90^\circ = 1\)

\(\Rightarrow n_1 = \frac{1}{\sin \theta_2} = \frac{1}{\sin 60^\circ} = \frac{2}{\sqrt{3}} \approx 1.15\)
Problem II (25 pts): A broad beam of light of wavelength 600 nm is incident at 90° on a thin, wedge-shaped film of index of refraction 1.50.

(a) If an observer views the light transmitted through this wedge, at the extreme left does the observer see a maximum or a minimum? Why?
- \( \frac{\lambda}{2} \) phase shift at top surface
- No phase shift due to path length at far left and (t=0)
⇒ Reflected rays interfere destructively.

However, by conservation of energy, a reflection minimum corresponds to a transmission maximum.

(b) If there are a total of 9 minima seen by the observer and one of these is centered on the extreme right of the wedge, how thick is the wedge at this point?

Transmission minima correspond to reflection maxima.

Reflection maxima: \( 2t = (M + \frac{1}{2})\lambda_n \)
⇒ First transmission minimum occurs for \( M = 0 \) (\( t = \frac{\lambda_n}{4} \))

Since there are 9 minima total, \( M = 0, 1, 2, 3, 4, 5, 6, 7, 8 \)

\[ 2t_{\text{max}} = (8 + \frac{1}{2})\lambda_n \]
⇒ \( t_{\text{max}} = \frac{(8.5)(600 \times 10^{-9})}{2(1.5)} \)
\[ = 1.7 \times 10^{-6} \text{ m} \]
Problem II (25 pts): A broad beam of light of wavelength 600 nm is incident at 90° on a thin, wedge-shaped film of index of refraction 1.50.

(a) If an observer views the light transmitted through this wedge, at the extreme left does the observer see a maximum or a minimum? Why?

- \( \lambda \) phase shift at top surface
- no phase shift due to path length when thickness is zero

\[ \Rightarrow \text{Reflected rays are out of phase when } t=0 \]

\[ \Rightarrow \text{Observer sees a minimum in the reflected light} \]

(b) If there are a total of 9 minima seen by the observer and one of these is centered on the extreme right of the wedge, how thick is the wedge at this point?

\[ \min \Rightarrow M=8 \text{ since there are 4 minima} \]
\[ m=0, 1, 2, 3, 4, 5, 6, 7, 8 \]

Reflected minima: \( t = M\lambda_n \Rightarrow t \text{ is max when } m \text{ is max} \)

\[ t_{\text{max}} = \frac{8\lambda}{2n} = \frac{8(600 \times 10^{-9} \text{m})}{2(1.5)} = 1.6 \times 10^{-6} \text{m} \]
Problem III (25 pts): Consider the object and two lenses shown in the figure.

(a) Where is the image of the first lens located relative to the first lens?
\[
\frac{1}{P_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow \frac{1}{i_1} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow i_1 = +20\text{cm}
\]

20 cm to right of first lens.

\[
\frac{w_1}{P_1} = \frac{i_1}{f_1} = -\frac{20\text{cm}}{20\text{cm}} = -1
\]

(b) Where is the final image of the system located relative to the location of the second lens?
\[
\frac{1}{P_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow \frac{1}{i_2} = \frac{1}{-20} - \frac{1}{5} = -\left(\frac{1}{20} + \frac{1}{5}\right) = -\left(\frac{1}{20} + \frac{4}{20}\right) = -\frac{1}{4}
\]

\[
i_2 = -4\text{cm}
\]

4 cm to left of second lens.

(c) What is the overall magnification of the system?
\[
\nu_2 = \frac{-i_2}{P_2} = -\frac{-4\text{cm}}{5\text{cm}} = \frac{4}{5}
\]

\[
u = \nu_1 \cdot \nu_2 = (-1)(\frac{4}{5}) = -\frac{4}{5}
\]

(d) Is the final image real or virtual?

Virtual
Consider a double slit experiment for which $\lambda = 550 \text{ nm}$, $d = 0.150 \text{ mm}$, and $a = 30 \mu\text{m}$.

(a) At what angle (in radians) does the first minimum in the diffraction envelope occur?

$$\alpha \sin \theta = m \lambda$$

$$m = 1$$

$$\Rightarrow \sin \theta = \frac{\lambda}{\alpha}$$

$$\Rightarrow \theta = \sin^{-1} \left( \frac{550 \times 10^{-9}}{30 \times 10^{-6}} \right) = 1.05^\circ = 0.0183 \text{ rad}.$$ 

(b) How many bright fringes appear between the first diffraction-envelope minima to either side of the central maximum?

$$d \sin \theta = \frac{m \lambda}{\alpha}$$

$$m = \frac{d \sin \theta}{\lambda} = \frac{0.15 \times 10^{-3}}{30 \times 10^{-6}} = 5.$$ 

$$\therefore m = 5 \text{ fringe is on the diffraction minimum.}$$

(c) What is the ratio of the intensity of the third bright fringe to the intensity of the central fringe?

$$\frac{I_3}{I_m} = \left( \frac{\sin \frac{3\lambda}{\alpha}}{\sin \theta} \right)^2$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$ 

$$d \sin \theta = 3 \lambda \Rightarrow \sin \theta = \frac{3 \lambda}{d}$$

$$\Rightarrow \alpha = \frac{\pi a}{\lambda} \cdot \frac{3 \lambda}{d} = \frac{3 \pi a}{d}.$$ 

$$\frac{I_3}{I_m} = \left( \frac{\sin \frac{2 \pi \times 30 \times 10^{-6}}{0.15 \times 10^{-3}}}{\sin \frac{\pi}{30 \times 10^{-6} / 0.15 \times 10^{-3}}} \right)^2 = \left( \frac{\sin \frac{\pi}{d}}{\frac{\pi a}{d}} \right)^2 = 0.255.$$