Name (1 pt): ____________________________

Recitation Instructor (1 pt): ____________________________

There are four pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

Problem I.1 (10 pts) A thin soap film of index of refraction \( n = 1.5 \) is suspended in air. If it is illuminated with light of wavelength \( \lambda = 500 \text{ nm} \), what is the minimum thickness for which light reflected from each of its surfaces is in phase?

\[
\begin{align*}
\chi_1 & = \frac{\lambda}{2n} \\
\chi_2 & = \frac{\lambda}{2n_2} \\
\chi_1 + \chi_2 & = \frac{\lambda}{n} \\
\end{align*}
\]

\[
\chi_1 + \chi_2 = \frac{\lambda}{n} = \frac{500 \text{ nm}}{1.5} \approx 333 \text{ nm}
\]

Problem I.2 (10 pts) Two flat pieces of plastic of index of refraction \( n_2 = 1.4 \) and \( n_3 = 1.6 \) are stacked atop one another, with air both above and below the stack. If a light ray is incident on the top piece of plastic with an angle relative to the normal of 30°, what will be the angle relative to the normal that the ray exits the lower piece of plastic?

\[
\begin{align*}
n_1 \cdot \sin \theta_1 & = n_2 \cdot \sin \theta_2 \\
n_2 \cdot \sin \theta_2 & = n_3 \cdot \sin \theta_3 \\
n_3 \cdot \sin \theta_3 & = n_4 \cdot \sin \theta_4 \\
n_4 \cdot \sin \theta_4 & = n_5 \cdot \sin \theta_5 \\
n_5 & = n_4 \\
\end{align*}
\]

\[
\theta_5 = \theta_4 \approx 30°
\]
Problem II (25 pts): Consider the object and two lenses shown in the figure.

(a) If there were no second lens, what would be the location of the first image relative to the first lens?

\[
\frac{1}{p} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow \frac{1}{i_1} = \frac{1}{-20} - \frac{1}{-20} = -\frac{1}{10} \quad \text{so} \quad i_1 = -10 \text{cm}
\]

(b) Where is the location of the final image relative to the second lens?

\[
\frac{1}{p} + \frac{1}{f_2} = \frac{1}{i_2} \Rightarrow \frac{1}{i_2} = \frac{1}{-20} - \frac{1}{20} = -\frac{1}{10} \quad \text{so} \quad i_2 = -10 \text{cm}
\]

(c) What is the overall magnification of the system?

\[
m = \frac{-i_2}{-20} = \frac{1}{2}
\]

\[
m = m_1m_2 = \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) = \frac{1}{4} \quad \text{so} \quad M = \frac{1}{4}
\]

(d) Is the final image real or virtual?

\[\sqrt{\text{virtual}}\]
Problem III (25 pts)
Consider a double slit problem for which the slit width is $a$ and the slit separation $d$. If the light has a wavelength of 400 nm and the central maximum of the diffraction pattern has a total angular width (i.e., the distance between the minima on either side of it) of 0.01 radians.

(a) What is the slit width?

\[ \lambda = 400 \text{ nm} \]

\[ a \sin \theta = \lambda \]

\[ \sin \theta \approx \theta \text{ for small } \theta \]

\[ \Rightarrow a \theta \approx \lambda \Rightarrow a \approx \frac{\lambda}{\theta} = \frac{400 \text{ nm}}{0.005} = 80 \mu\text{m} \]

(b) If the forth minima in the interference pattern is located at 0.004 radians, what is the slit separation?

\[ \text{interference minima} \]

\[ m = 0 \text{ is 1st min} \]

\[ m = 3 \text{ is 4th min} \]

\[ d \approx \frac{(m + \frac{1}{2}) \lambda}{\theta} \]

\[ = \frac{(3 + \frac{1}{2})(400 \text{ nm})}{0.004} = 350 \mu\text{m} \]

(c) What is the path difference for the light from each of the two sources that combines to form the fourth maximum in the interference pattern?

\[ \text{path difference} = d \sin \theta = m \lambda \]

If you count $m = 0$ (central fringe) as 1st fringe,

\[ \text{path diff} = 3 \lambda = 1200 \text{ nm} = 1.2 \mu\text{m} \]

If you count $m = 1$ as first fringe,

\[ \text{path diff} = 4 \lambda = 1600 \text{ nm} = 1.6 \mu\text{m} \]
Problem IV (25 pts):

A light ray enters a glass slab at point A and then undergoes total internal reflection at point B. What is the minimum value for the index of refraction of the glass that can be inferred from this information? Air surrounds the glass.

\[ A: \quad n_1 \sin \theta_1 = n_2 \sin 45^\circ \]

\[ B: \quad n_2 \sin (90^\circ - \theta_2) = n_1 \sin \theta_1 \]

\[ \sin (90^\circ - \theta_2) = \cos \theta_2 \]

\[ n_2 \cos \theta_2 = 1 \]

\[ n_2 \sin \theta_2 = \sin 45^\circ \]

\[ \frac{n_2 \sin \theta_2}{n_2 \cos \theta_2} = \frac{\sin 45^\circ}{1} \]

\[ \tan \theta_2 = \sin 45^\circ \]

\[ \theta_2 = \tan^{-1} (\sin 45^\circ) \]

\[ \theta_2 = 35.26^\circ \]

\[ \sin 45^\circ = n_2 = 1.22 \]

\[ \frac{\sin 45^\circ}{\sin 35.26^\circ} \]