A block rides on a piston that is moving vertically with simple harmonic motion.

a) If the SHM has a period of 1.0 s, at what amplitude of motion will the block and piston separate?

Block and piston will separate when $a_{\text{max}} \geq g$.

For, SHM $a(t) = -\omega^2 x_m \cos(\omega t + \phi)$

Separation will begin when $a_{\text{max}} = g$

$\Rightarrow \omega^2 x_m = g$  $\Rightarrow x_m = \frac{g}{\omega^2}$

$\Rightarrow x_m = \frac{g}{(\frac{2\pi}{T})^2} = \frac{T^2 g}{4\pi^2} = \frac{(9.8 \text{ m/s}^2)(1.0 \text{ s})}{4\pi^2} = \frac{0.25 \text{ m}}{\sqrt{4\pi^2}}$

$= 0.25 \text{ m}$

b) If the piston has an amplitude of 5.0 cm, what is the maximum frequency for which the block and piston will be in contact continuously?

Same procedure as a) leads to

$\omega_m^2 x_m = g$  $\Rightarrow (\omega_m f_m)^2 x_m = g$

$\Rightarrow f_m^2 = \frac{g}{(\omega_m)^2 x_m}$  $\Rightarrow f_m = \frac{1}{\omega_m} \sqrt{\frac{g}{x_m}}$

$= \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{0.05 \text{ m}}} = \sqrt{2.2 \frac{1}{5}}$

$\omega_m = 2\pi f_m = 14 \frac{1}{5}$