7.1 The student Misty Fide comes to you and says, "I am confused by tunneling. Consider the rectangular potential barrier of height V_0 between $-a/2$ and $a/2$. When $E < V_0$ we know that the wave function in the classically forbidden region is a linear combination of $\exp(-\kappa x)$ and $\exp(+\kappa x)$ with κ real. Now I remember that the probability current density vanishes when the wave function is real. There is an incident current density and there is a transmitted current density because of tunneling. How can there be a current density for $|x| > a/2$ without a current density in between? This quantum mechanics sure is weird." Provide a lucid explanation for this student with an explicit calculation. (5 points)

7.2) **Propagating waves in periodic potentials exhibit bands:**
The solution was sketched in class. Look at the infinite LC-ladder in the accompanying figure. I suggest that you simplify the boundary conditions by extending the ladder to $-\infty$ and study the propagation of waves. Show that there is an allowed band of frequencies and find the upper bound on the propagating mode frequencies. (6 points)

Find an analogous **mechanical** system with elementary components. (2 points)

Consider $2N$(even) point particles in one dimension connected by springs. The spring constants alternate in strength with values α_1 and α_2. You may assume that all the particles have the same mass m for algebraic simplicity. It is convenient to divide the $2N$ particles into N cells each with two particles and denote the displacement of the two particles (from equilibrium) in the j^{th} cell by $u_1(j)$ and $u_2(j)$ respectively. Write down the classical equations of motion for the displacements from equilibrium making sure that even and odd sites have different behavior.[1] Use periodic boundary conditions and find the allowed frequencies and wave vectors. How many bands do you obtain? When quantized these normal modes become **phonons**. (8 points)

[1]Hint: The equation of motion for one of them will have the form

$$\ddot{u}_1(ja) = -\alpha_1[u_1(ja) - u_2(ja)] - \alpha_2[u_1(ja) - u_2((j-1)a)]$$

where I have assumed the spring constant within the cell is α_1 and that between the cells is α_2. Yo may have them interchanged but that is immaterial. There is a similar equation for $u_2(ja)$.

Periodic optical nanostructures lead to bands for light (electromagnetic waves) and are called **photonic** crystals. Lord Rayleigh had worked out photonic band gaps in one dimension in the nineteenth century. By considering periodic dielectric functions one can study photonic bands of electromagnetic waves. I resisted the temptation of making you work out the band structure of light.

7.3) Exercise 7.4.2, 7.4.5, and 7.4.6. Please read pages 203-212 and do these whether or not they are covered in class. They involve simple algebraic manipulations, albeit very useful. (6 points)

7.4) Consider a perfectly flexible string with mass density λ under a tension T. Let $\psi(x,t)$ be the transverse displacement at a position x at time t. You may assume that the equation of motion is

$$\frac{\partial^2\psi(x,t)}{\partial t^2} = \frac{T}{\lambda}\frac{\partial^2\psi(x,t)}{\partial x^2}\,.$$

It is good to review the derivation for your own edification but you need not submit it. Consider an infinite string and check that $Ae^{ikx-i\omega t}$ yields a traveling wave solution and find the dispersion relation. Of course using the complex exponential is for mathematical convenience and there are no imaginary parts to a classical string.

Next consider an infinite string with a discontinuity in the mass density λ at $x = 0$: λ_1 for $x < 0$ and λ_2 for $x > 0$. State clearly the boundary conditions satisfied by $\psi(x,t)$ at $x = 0$. For a wave incident from the left with amplitude A_I (coefficient of $e^{ikx-i\omega t}$) find the reflected and transmitted amplitudes denoted by A_R and A_T. Find A_T/A_I and A_R/A_I and compare your answers with those for the potential step in one-dimensional quantum mechanics.

If $\lambda_2 > \lambda_1$ draw pictures for a localized wave incident from the left and the shapes of the transmitted and reflected wave forms. (10 points)