Name: ______________________________

Physics 261: Worksheet #3

This worksheet is part of the homework for this week. It illustrates one possible method for solving a system of N linear equations with N unknowns ("linear" here means that each variable appears as a first power only). Such systems arise repeatedly in Newton's law problems.

The method is based on successive elimination of variables. If you get stuck, find someone to discuss it with or send email to one of the instructors. **Don't forget the back!**

Basic plan:

1. Pick a variable to eliminate and solve for it in one of the equations.
2. Replace that variable in *all* of the remaining equations.
3. With the remaining equations, repeat steps 1 and 2. Stop when only one equation remains.
4. Substitute in reverse order to determine all of the unknowns in terms of knowns.

Example: [K&K 2.15]. Goal: Find T. The unknowns are T, $\ddot{x}_1$, $\ddot{x}_2$, and $\ddot{y}_3$. The equations are:

$$T - \mu m_1 g = m_1 \ddot{x}_1 \qquad (a) \qquad\qquad m_3 g - 2T = m_3 \ddot{y}_3 \qquad (c)$$

$$T - \mu m_2 g = m_2 \ddot{x}_2 \qquad (b) \qquad\qquad 2\ddot{y}_3 - \ddot{x}_1 - \ddot{x}_2 = 0 \qquad (d)$$

Since we're looking for T, we'll eliminate the other variables in turn, repeating steps 1 and 2 three times.

1. $(a) \Rightarrow \ddot{x}_1 = \dfrac{T - \mu m_1 g}{m_1}$ 2. $(d) \Rightarrow 2\ddot{y}_3 - \dfrac{T - \mu m_1 g}{m_1} - \ddot{x}_2 = 0 \qquad (d')$

1. $(b) \Rightarrow \ddot{x}_2 = \dfrac{T - \mu m_2 g}{m_2}$ 2. $(d') \Rightarrow 2\ddot{y}_3 - \dfrac{T - \mu m_1 g}{m_1} - \dfrac{T - \mu m_2 g}{m_2} = 0 \qquad (d'')$

1. $(c) \Rightarrow \ddot{y}_3 = \dfrac{m_3 g - 2T}{m_3}$ 2. $(d'') \Rightarrow 2\dfrac{m_3 g - 2T}{m_3} - \dfrac{T - \mu m_1 g}{m_1} - \dfrac{T - \mu m_2 g}{m_2} = 0 \qquad (d''')$

Now factor T in (d''') and solve for T:

$$\Longrightarrow \quad -T\left(\frac{4}{m_3} + \frac{1}{m_1} + \frac{1}{m_2}\right) + 2\mu g + 2g = 0 \qquad\qquad \Longrightarrow \quad T = \frac{2g(\mu + 1)}{\dfrac{4}{m_3} + \dfrac{1}{m_1} + \dfrac{1}{m_2}}$$

Find $\ddot{x}_1$, $\ddot{x}_2$, and $\ddot{y}_3$ by substituting T into (a), (b), and (c) [step 4].

Now you try ...

Practice Problem A: Solve for $\ddot{x}_2$ and T. [Note: These are just made-up equations!]

$$2T - \mu m_1 g = m_1 \ddot{x}_1 \quad (a) \qquad m_2 g - T = m_2 \ddot{y}_2 \quad (c)$$

$$T - \mu m_2 g = m_2 \ddot{x}_2 \quad (b) \qquad \ddot{y}_2 - 2\ddot{x}_1 - \ddot{x}_2 = 0 \quad (d)$$

Practice Problem B: Solve for x, y, and z. Make sure to *check* your answer by substituting back into the original equations!

$$3x + 2y - z = -2 \quad (a) \qquad x - y + 2z = 3 \quad (b) \qquad 2x + 3y - z = 1 \quad (c)$$