Physics 263: MATLAB Cheatsheet VII

This sheet summarizes some matrix manipulations in MATLAB. We’ll start with a cookbook description of solving a matrix equation and then fill in some more details.

1. Solving Matrix Equations

Suppose we want to solve the simultaneous equations

\[
\begin{align*}
3x - 2y &= 17 \\
5x + 3y &= 3
\end{align*}
\]

We write this in the form \(MX = B\), with \(M\) a matrix and \(B\) a column vector, then find the desired column vector \(X\) from \(X = M^{-1}B\) using the MATLAB \texttt{inv} function:

\[
\begin{align*}
\text{>> } M &= \begin{bmatrix} 3 & -2 \\ 5 & 3 \end{bmatrix} & \% \text{ separate the rows of the matrix with ;'s} \\
\text{M} &= \\
& \begin{bmatrix} 3 & -2 \\ 5 & 3 \end{bmatrix} \\
\text{>> } B &= \begin{bmatrix} 17 \\ 3 \end{bmatrix} & \% \text{ note the ; to make it a column vector (two rows)} \\
\text{B} &= \\
& \begin{bmatrix} 17 \\ 3 \end{bmatrix} \\
\text{>> } X &= \text{inv}(M)B & \% \text{ regular matrix multiplication uses * (not .*)} \\
\text{X} &= \\
& \begin{bmatrix} 3.0000 \\ -4.0000 \end{bmatrix}
\end{align*}
\]

That’s the answer: \(x = 3\) and \(y = -4\). \textit{Note:} If you had defined the vector \(B\) as a row vector instead of a column vector, you would have received an error like this:

\[
\begin{align*}
\text{>> } B_{\text{row}} &= \begin{bmatrix} 17 & 3 \end{bmatrix} & \% \text{ there is no ";" so this is a row vector} \\
\text{B}_{\text{row}} &= \\
& \begin{bmatrix} 17 & 3 \end{bmatrix} \\
\text{>> } X &= \text{inv}(M)B_{\text{row}} \\
\text{??? Error using ==> mtimes} \\
& \text{Inner matrix dimensions must agree.}
\end{align*}
\]

An alternative way to using \texttt{inv} to solve the equation is:

\[
\begin{align*}
\text{>> } X &= M\backslash B & \% \text{ The "\" means "matrix left division"} \\
\text{which is actually preferred because it solves the problem much more efficiently.}
\end{align*}
\]
2. Creating Vectors and Matrices and Accessing Elements

Both vectors and matrices are specified by entries between [ ]’s with semicolons; used to separate rows. So

\[
A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -5 & 4 \\ -1 & 3 & -2 \end{bmatrix} \quad B = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \quad C = \begin{pmatrix} 3 & 4 & 5 \end{pmatrix}
\]

are entered as

\[
\text{>> A} = [1 -2 1; 2 -5 4; -1 3 -2]
\]

\[
A =
\begin{bmatrix}
1 & -2 & 1 \\
2 & -5 & 4 \\
-1 & 3 & -2
\end{bmatrix}
\]

\[
\text{>> B} = [3; 4; 5]
\]

\[
B =
\begin{bmatrix}
3 \\
4 \\
5
\end{bmatrix}
\]

\[
\text{>> C} = [3 4 5]
\]

\[
C =
\begin{bmatrix}
3 & 4 & 5
\end{bmatrix}
\]

3. Matrix Operations

a. **Matrix multiplication.** Ordinary matrix multiplication is performed by using *. In contrast, .* is used for element-by-element operations (e.g., \(A*B\) is matrix multiplication while \(A.*B\) multiplies each element in \(A\) by the corresponding one in \(B\)).

b. **Inverse of a matrix.** The inverse of the square matrix \(A\) is designated \(A^{-1}\) and is defined by \(AA^{-1} = A^{-1}A = I\), where \(I\) is the identity matrix. We can find the inverse of a matrix either by raising \(A\) to the \(-1\) power, i.e., \(A^{-1}\), or with the \text{inv}(A)\) function.

c. **Determinant of a matrix.** The \text{det} function returns the determinant of a square matrix. That is, \text{det}(A)\) gives the determinant of the matrix \(A\).

4. Special Matrices

a. For a 3 \times 3 unit matrix, use \text{eye}(3) while for \(N \times N\) use \text{eye}(N).

b. \text{zeros}(N)\) is an \(N \times N\) matrix of zeros while \text{zeros}(1,N)\) is an \(N\)-dimensional row vector of zeros and \text{zeros}(N,1)\) is an \(N\)-dimensional column vector of zeros.

c. \text{ones}(N)\) is an \(N \times N\) matrix of ones while \text{ones}(1,N)\) is an \(N\)-dimensional row vector of ones and \text{ones}(N,1)\) is an \(N\)-dimensional column vector of ones.