Physics 263: Chapters 3/4/5 Practice Problems

These are practice for Chapter 3 problems of integration in multiple variables plus problems for Chapter 4 on the (absolute) convergence of series plus the first problems from Chapter 5 on complex numbers.

Three-Dimensional Integrals

1. Find the moment of inertia of a sphere of radius \( R \) and mass \( M \) with a uniform density in cylindrical, spherical, and cartesian coordinates.

2. Repeat the last problem but now with a density that varies like \( \rho(x) = \rho_0 \cos^2 \theta \) (so you need to find the mass as well to express \( \rho_0 \) in terms of \( M \)).

3. Repeat one more time with \( \rho(x) = \rho_0(1 - r/R) \).

Convergence of Power Series

Find the radius of convergence \( R \) for each of these functions or series by the ratio test.

1. \( \sum_{i=1}^{\infty} \frac{x^n}{3^n} \)

2. \( \sum_{i=1}^{\infty} nx^n \)

3. \( \sum_{i=1}^{\infty} \frac{(x+1)^n}{n^2} \)

4. \( \cosh x \)

5. \( \sin x \)

6. \( (1 + x)^p \)
Complex Numbers

Here are some basic exercises like the ones in BTM chapter 5:

1. Solve for $x$ and $y$ given

$$
\frac{1 + 2i}{3 - 4i} + \frac{3}{x - iy} = 8 - 2i.
$$

For each of the following numbers, find the real part, imaginary part, modulus (i.e., the magnitude), the complex conjugate, and the inverse (i.e., given $z$, find $1/z$):

1. $\frac{5}{8 + 6i}$
2. $(2 - 3i)^3$
3. $(1 + i)^4$
4. $\frac{2 + i}{z - i}$
5. $\frac{1 + \sqrt{5}i}{1 - \sqrt{2}i}$

For each of the following pairs of numbers, give their polar form, their complex conjugates, their moduli, product, the quotient $z_1/z_2$, and the complex conjugate of the quotient:

1. $z_1 = \frac{1 + 2i}{\sqrt{5}} \quad z_2 = \sqrt{2} + i$

2. $z_1 = \frac{1 + 2i}{1 - 2i} \quad z_2 = \left[\frac{1 - 3i}{1 + i}\right]^2$