Physics 263: Chapter 8 Practice Problems I

These are practice for the first part of Chapter 8 of BTM, which covers matrices and determinants.

2 × 2 Matrix Review

If we have a 2 × 2 matrix $M$ written as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

then the inverse $M^{-1}$ and the determinant $|M|$ are given by

$$M^{-1} = \frac{1}{|M|} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \quad \text{where} \quad |M| = M_{11}M_{22} - M_{12}M_{21}.$$

1. The rotation matrix $R_\theta$, which rotates a vector with components $(x, y)$ by angle $\theta$ to the vector $(x', y')$, is given by

$$R_\theta = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

Verify that $R_{\theta+\theta'} = R_{\theta'} R_\theta$. What is the determinant of $R_\theta$?

2. Solve the following system of equations by computing a matrix inverse $[x = 3, y = -4]$:

$$3x - 2y = 17$$
$$5x + 3y = 3$$

3. Why is there not a unique solution to the following equations? What is the value of the appropriate determinant?

$$6x + 9y = 3$$
$$2x + 3y = 1$$
Larger Matrices

1. Solve the following simultaneous equations by matrix inversion:

\[
\begin{align*}
2x + 3y - 4z &= -3 \\
3x - 2y + 5z &= 24 \\
x + 4y - 3z &= -6
\end{align*}
\]

2. Show that

\[
\begin{bmatrix}
2 & 1 & 3 \\
0 & 1 & 2 \\
-1 & 1 & 1
\end{bmatrix}
^{-1} =
\begin{bmatrix}
1 & -2 & 1 \\
2 & -5 & 4 \\
-1 & 3 & -2
\end{bmatrix}
\]

3. Find the cofactor matrix and the inverse for the matrix

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 10
\end{bmatrix}
\]

Verify that your inverse does what it is supposed to do.

4. Does the following system have a unique solution?

\[
\begin{align*}
x + 3y - 2z &= 4 \\
2x - y + z &= 3 \\
4x - 9y + 7z &= 1
\end{align*}
\]