Physics 263: BTM Problem Set #7

Although this is called “BTM Problem Set #7”, none of the problems are actually from BTM, although they are clear extensions. They were selected from standard texts as further practice of an important class of problems: integration over three-dimensional volume elements. Please ask questions! It is due by 5:30pm in the box in 1011 on Friday, April 21.

1. **Rotating cube.** Find the moment of inertia of a uniform cube of mass $M$ with sides of length $L$ about an axis through the centers of two opposite sides.

2. **Solid cone.** Find the mass and moment of inertia about the symmetry axis of a cone of height $h$ and base radius $R$. The density of the cone varies linearly with the height according to
   \[ \rho(z) = \rho_0 (1 + z/h) , \]
   where $z = 0$ corresponds to the base of the cone. Express your answers in terms of $h$, $R$, and $\rho_0$.

3. **Charged hemisphere.** A hemisphere of radius $R$ made of an insulating solid has a variable charge density $\rho_{\text{ch}}(r)$. If the hemisphere has its flat side centered on the $x$-$y$ plane, then $\rho_{\text{ch}}(r)$ is given by
   \[ \rho_{\text{ch}}(r) = \rho_0 \frac{r \cos \theta}{R} \]
   for $r \leq R$ and zero for $r > R$. Find the total charge in terms of $\rho_0$ and $R$.

4. **BONUS: Exponential sphere.** Find the moment of inertia of a sphere of radius $R$ and mass $M$ about an axis through the center if the density (mass per volume) varies as
   \[ \rho(r) = \rho_0 e^{-kr/R} . \]
   for $r \leq R$ (and is zero for $r > R$). Express your answer in terms of $R$, $M$, and $k$. Show that the limit $k \ll 1$ makes sense.