Physics 263: MATLAB Assignment #1

This assignment is basically the third part of 1094 Session 2, on “Numerical Derivatives and Round-Off Error.” It is due by midnight on Thursday, April 21. Turn the assignment in by emailing the answers to the questions with your program and graph attached, to Daniel Bibireata at db@mps.ohio-state.edu.

You may want to refer back to the “MATLAB Cheatsheets” for help. These are available from the 263 homepage.

1. Run deriv_test.m by typing deriv_test at the >> prompt and enter 2 for x0. The resulting plot is of the error made by using approximation 1 for the derivative of a test function (see the “Cheatsheet” for the list of approximations from class). What is the slope of the line? Is that consistent with how the error should scale with ∆x?

2. Find deriv_test.m and test_function1.m in the directory listing and bring them up in the “Editor”. Along with “MATLAB Cheat Sheet II” there are printouts of these files. Step through the listings and figure out how they work. What is the test function?

3. Modify deriv_test.m so that the lower limit of Delta x is 10^{-20} and then run deriv_test again. Now there are three regions in the plot. How would the graph be different if you could take the true mathematical limit of ∆x going to zero on the computer?

4. What you are seeing is evidence for “round-off error” in the calculation. Because decimal numbers are represented with a finite number of binary digits (0’s and 1’s), there is a limit on how close two numbers can get and still be represented accurately. This limit is related to the ”machine precision”. It means that when Delta x gets too small, the calculated difference between the function at x0 and x0 + ∆x gets less and less precise. Why would this lead to the change in slope that you observe below 10^{-5}? What is the optimum choice for ∆x for this problem?

5. Explain what is probably happening at still smaller ∆x.

6. Modify deriv_test.m to add a second plot to the graph that calculates the approximate derivative using method 3. This is the program and graph that you need to turn in. How do the slopes compare? What do you conclude?

7. Copy test_function1.m to test_function2.m and try another function. Do you get the same (or similar) results?
8. **Bonus Problem.**

(a) Derive an expression for an approximation to the second derivative of $f(x)$ from the Taylor expansions for $f(x + \Delta x)$ and $f(x - \Delta x)$.

(b) Copy and modify the `deriv_test.m` code to implement this expression for the test function (which is set up to return the exact second derivative if requested). Derive how you expect the error should scale with $\Delta x$ and compare to what you find from your program. Send in the graph and your program.