Plan for today: Work through session 8.

The basic problem is a forced (driven), nonlinear, oscillator.

e.g., \( F(x) = \begin{cases} -kx^3 & x > 0 \\ +k|x|^{\alpha} & x < 0 \end{cases} \) (no damping yet)

\[ F = ma \Rightarrow \frac{d^2x}{dt^2} = \frac{dV}{dx} = F_k(x) + F_{ext}(x,t); \quad v = \frac{dx}{dt} \]

So, \( y^{(0)}(t) = x(t) \quad y^{(1)}(t) = v(t) \quad y^{(2)}(t) = \frac{dx}{dt} \)

\[ \Rightarrow \frac{dy^{(0)}}{dt} = y^{(1)} \quad \frac{dy^{(2)}}{dt} = \frac{1}{m} F_k(x) + \frac{F_{ext}}{m} \]

5 equations, the initial conditions

\[ y^{(0)}(t=0) = x_0 \quad y^{(1)}(t=0) = v_0 \]

Things to note:

- Total energy \( E(t) = KE(t) + PE(t) = \frac{1}{2}mv(t)^2 + V(x(t)) \)
- What do you expect this to look like?
- What if damped?

- Virial Theorem: \( \langle KE \rangle = \frac{1}{2} \langle PE \rangle \) How to check?

- "Antilinear" oscillator means \( p \neq 2 \). What is special about a harmonic oscillator? (e.g. period vs. amplitude)