Wednesday 7:00-20 Class

- Finite element problem.
  - What to do with a discontinuous boundary condition?
    - My solution: In triangle 1, \( A = 100 \)
    - In triangle 0, \( A = 0 \).
    - Other ideas?
  - A major flaw of the triangle check routine is revealed when points are on the sides of triangles.
    - Uses: if (double a == double b)
      - almost always a bad idea!
    - alternative: if fabs(a-b) < tolerance
      - for a small tolerance
      - check if no triangle is assigned.

- Assignment #2
  - Practice the Qt basics a lot more (finishing session 16)
  - You can substitute bonus problems.
    - Eg. random walk
    - GPlanner widget
    - (I haven't done this.)

- Today (and Monday)
  - Solving integral equations in the context of reproducing results from a paper.
  - I give you much less and want you to plan a solution, more than just solving the equation.
What you should take from the "Numerical Recipes" discussion.

If my equation is
\[ g(t) = \int_{a}^{b} K(t,s) f(s) \, ds \]
known \( a \) \( b \) known
and we want to find \( f(s) \), this can be thought of as a
matrix equation
\[ \hat{L} \cdot \hat{f} = \hat{q} \]

We can make it a matrix equation by dividing \([a, b]\) into
a grid: \( s_0, s_1, \ldots, s_n \) and using an integration formula
like Trapezoid rule, Simpson's rule, or Gaussian quadrature.

\[ \Rightarrow \begin{align*}
  g(t) &\rightarrow g_i = g(t_i) \\
f(s) &\rightarrow f_j = f(s_j) \\
K(t, s) &\rightarrow K_{ij} = K(t_i, s_j)
\end{align*} \]

\[ \Rightarrow \sum_{j=1}^{n} K_{ij} f_j g_j = g_i \quad \Rightarrow \text{standard matrix equation} \]
\[ \text{for } i = 1, N \quad \text{(unique solution if } g(t) \text{ and } K \text{ is invertible)} \]

What if
\[ f(t) = \int_{a}^{b} K(t,s) f(s) \, ds + g(t) \]

What is the matrix equation to solve?
\[ \begin{align*}
  f_i &= \sum_{j=1}^{n} K_{ij} f_j + g_i \\
  \Rightarrow \sum_{j=1}^{n} (K_{ij} - S_{ij}) f_j &= -g_i
\end{align*} \]
Solving numerically:

- can use any quadrature, but if smooth and nonsingular,
  Gaussian quadrature is best.

\[
\int_a^b h(x) \, dx = \sum_{i=1}^n h(x_i) \, w_i \quad \text{where } x_i, w_i \text{ are calculated once at the beginning.}
\]

To evaluate \( f(t) \) at \( t + t_i \), use

\[
f(t) = \sum_{j=1}^N w_j K(t, s_j)f(s_j) + g(t)
\]

once you have found \( f(s_j) \).

Another method: iteration for \( f(t_i) = \int_{a}^{b} K(t, s) f(s) \, ds + g(t) \)

\[
f(t_i) = \sum_{j=1}^n K(t_i, s_j)f(s_j) + g(t_i)
\]

\[
f_{old}(t_i) = g(t_i)
\]

\[
f_{new}(t_i) = \sum_{j=1}^n K(t_i, s_j)f_{old}(s_j) + g(t_i) \quad \text{repeat until}
\]

changes are small.

- can use "fraction"
4/21/04

Quicks overview of Casper et al. paper.

- N spin-1/2 fermions in a one dimensional box of length L.
- Interact with S-function potentials.
- Continuum limit of Hubbard model in 1-d.

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} - \sum_{i<j}^{N} \delta(x_i - x_j) \quad \delta_{ij} > 0 \]

- Density if \( \rho = N/L \).
- Goal: find the energy/ground state for the ground state.

For two particles, \( E_{1}(\mathbf{r}) = -\frac{m_0^2}{4\hbar^2} \quad \) (Just solve S-equation).

If we scale \( x_i \) by \( \rho \), density \( x'_i = \rho x_i \), then

\[ H' = \frac{\rho H}{\rho^2} = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \frac{\partial^2}{\partial x'_i^2} - \sum_{i<j}^{N} \delta(x'_i - x'_j) \]

with \( \lambda = \frac{m_0}{\rho \hbar} \) dimensionless.

So solving for one \( \lambda \) gives result for all combinations of \( N \) and \( \rho \) that give the same \( \lambda \).

In Appendix A, write or Gaudin equations for each solution:

\[ E(\lambda) = \left( E_0(N)/N \right) / (E_0/2) \quad \text{for} \quad 0 \leq \lambda \leq \infty \]

Your job is to solve these equations.

\[ F_{\lambda}(y) = \int_{-\infty}^{\infty} \frac{dy}{1 + k^2(k-y)^2} \]

unknown function

\[ E(\lambda) = -1 + \frac{4}{\pi} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} [\delta(y) - F_{\lambda}(y)] \]

unknown, depends on \( \lambda \)