Relative Error for Model Partition Function

- $O(\lambda)$ pert.
- $O(\lambda^2)$ pert.
- $O(\lambda)$ asym.
- $O(\lambda^2)$ asym.

Breakdown of perturbation theory around $\lambda = 60 \approx 1$.

Corrections get worse, but still 10% accuracy even for large $\lambda$.

As $\lambda \to \infty$, exact $\approx \frac{\sqrt{5}}{9} \lambda^{-\frac{3}{4}} \approx 1.02 \lambda^{-\frac{3}{4}}$.

Leading asymptotic $\approx \frac{\sqrt{\pi}}{9^\frac{3}{4}} \lambda^{-\frac{3}{4}} \approx 0.91 \lambda^{-\frac{3}{4}}$.

Model Partition Function is:

$$Z_\lambda = \int_{-\infty}^{\infty} \frac{dx}{(2\pi)^{\frac{1}{2}}} e^{-\frac{\lambda x^2}{2} - \frac{\lambda^2 x^4}{4}}$$

evaluated here with $\alpha = 1$.
[Note: "relative error" in these plots is normalized to 1 at 0 terms.]