Nuclear Physics

1. Nuclei and strong interaction [Q12,Q13]
   - nuclear notation: $Z$ protons and $A$ total nucleons $\implies \frac{A}{2}X$; size: $r = r_0 A^{1/3}$, with $r_0 \approx 1.2$ fm
   - Strong interaction between color neutral objects (such as nucleons) is short range ($< 2$ fm)
   - Binding energy: $E_b = (\sum_{\text{parts}} E_{\text{parts}}) - E_{\text{sys}}$
   - Mass defect: $\Delta m = (\sum_{\text{parts}} m_{\text{parts}}) - m_{\text{sys}} = E_b/c^2 = Z m_H + N m_n - m_{\text{atom}}$
   - binding energy formula ($A > 20$): $E_b(A, Z) = a_I A - a_S A^{2/3} - a_C Z^2/A^{1/3} - a_A (A - 2Z)^2/A
     \text{ with } a_I = 15.56 \text{ MeV, } a_S = 17.23 \text{ MeV, } a_C = 0.697 \text{ MeV, } a_A = 23.285 \text{ MeV}$
   - binding energy per nucleon, $e_b = E_b/A$ and binding energy vs. mass ($E = mc^2$)

2. Radioactive decay (beta, alpha, gamma) [Q13,Q14]
   - Predicting stability and type of decay: energy level diagrams, binding energy formula, atomic mass
   - beta: $\beta^- \implies n \rightarrow p^+ + e^- + \nu_e \quad \beta^+ \implies p^+ \rightarrow n + e^+ + \nu_e \quad \text{EC} \implies p^+ + e^- \implies n + \nu_e$
   - alpha: helium nuclei (2 protons, 2 neutrons)
   - gamma: photons from decays of excited nuclear states
   - activity $\equiv -dN/dt = \lambda N \implies$ solution: $N(t) = N_0 e^{-\lambda t}$ $\implies$ half-life $t_{1/2} = \ln 2/\lambda$
   - $Z_{\text{stable}}(A)$ for stability against $\beta$ decay: $Z_{\text{stable}} = A \left( \frac{1.0084}{2 + 0.015 A^{-7/8}} \right)$
     \text{(This formula gives the stable isotope for even-odd nuclei ($A$ is odd) or even-even nuclei (both $Z_{\text{stable}}$ and
     $N$ are even). For odd-odd nuclei ($A$ even, $N$, $Z_{\text{stable}}$ odd) the neighboring even-even nuclei may have
     lower atomic mass, in which case the stable nuclei have $Z = Z_{\text{stable}} \pm 1$.)}
   - Beta decay arises from weak interactions; it leaves $A$ unchanged. $\beta^-$ decay converts a neutron to a proton, decreasing $N$ by 1 and increasing $Z$ by one. Electron capture (EC) and $\beta^+$ decay convert a proton in a neutron, increasing $N$ by 1 and decreasing $Z$ by one.
   - $\beta^-$ decay and EC from $i$ to $f$ are energetically possible if the atomic masses satisfy the relation $m_i > m_f$.
     $\beta^+$ decay is energetically possible if $m_i > m_f + 2m_e$.
   - Alpha decay is due to strong interactions. It arises from emission of a $^4$He nucleus, by tunneling through the Coulomb barrier. It changes $A$ by 4. Parent and daughter nuclei of decay chains involving both $\alpha$ and $\beta$ decays differ in mass number by multiples of 4.
   - Alpha decay is energetically allowed if $dE_b/dA \leq 7.07 \text{ MeV}$ where 7.07 MeV is the binding energy per nucleon in a $^4$He nucleus. For $\beta$-stable nuclei, this happens for $A$ larger than about 150.
   - Fission describes the decay of a large nucleus into two roughly equal-sized fragments. It additionally produces neutrons, since the fragment nuclei have smaller $A$ and prefer a lower $N/Z$ ratio for stability.\text{The extra neutrons can trigger further fission events, leading to a chain reaction.}
   - Decay rate $\lambda = dP_{\text{decay}}/dt = (1/N) dN_{\text{decay}}/dt$
     The activity $dN_{\text{decay}}/dt = \lambda N$ of a radioactive sample is measured in becquerel (1 Bq = 1 decay per second) or curie (1 Ci = $3.7 \cdot 10^{10}$ Bq).
   - Half-life $t_{1/2} = (\ln 2)/\lambda$
   - Exponential decay law: $N(t) = N(0) e^{-\lambda t}$
Laws of Thermodynamics

1. Zeroth Law of Thermodynamics: objects $A$ and $B$ are in thermal equilibrium if and only if $T_A = T_B$ [T1]
   - A thermometer is a device that quantifies temperature, calibrated to a standard temperature scale.
   - $T$ (in K) = $T$ (in °C) + 273.15 $\implies$ Use $T$ in kelvin in all formulas
   - $dU = mc \, dT$, where $m$ is the mass of the substance and $c$ is its specific heat.

2. First Law of Thermodynamics: $\Delta U = Q + W$ [T3]
   - $Q > 0$ if heat energy entering system; $W > 0$ if work energy entering system

3. Second Law of Thermodynamics: $\Delta S \geq 0$ [T5]
   - be able to explain in your own words why entropy increases

Ideal Gases [T2, T3]

1. Ideal gas law: $PV = Nk_BT$, where $N$ is the number of molecules and $k_B = 1.38 \times 10^{-23}$ J/K = 8.62 $^{-5}$ eV/K.
   - $n = \text{number of moles} = N/N_A$, where $N_A = 6.02 \times 10^{23}$.

2. “Equipartition theorem” [T2]
   - each degree of freedom gets $\frac{1}{2}k_BT$ on average $\implies$ translational $K = \frac{1}{2}m[\langle v^2 \rangle_{\text{avg}}] = \frac{3}{2}k_BT$
   - $v_{\text{rms}} = \langle [v^2]_{\text{avg}} \rangle^{1/2} = \sqrt{\frac{3k_BT}{m}}$
   - monatomic gas: $U = \frac{3}{2}Nk_BT$; diatomic gas: $U = \frac{5}{2}Nk_BT$
   - mode is fully unfrozen when $k_BT > 2\Delta E$ (c.f. Boltzmann factor $e^{-\Delta E/k_BT}$)

3. Gas processes [T3]
   - interpreting a $PV$ diagram $\implies$ ideal gas law applies at each point on curve
     - work from $W = - \int P \, dV$ $\implies$ area under curve
     - net work from closed cycle is enclosed area
     - find $T$ or $N$ from ideal gas law and $P$, $V$
     - find $\Delta U$ from $\Delta T$, then $Q$ from First Law
   - isochoric: $V$ = constant $\implies W = 0$
   - isobaric: $P$ = constant $\implies W = -P(V_f - V_i)$
   - isothermal: $T$ = constant $\implies W = - \int P \, dV = -(Nk_BT) \int dV/V = -Nk_BT \ln(V_f/V_i)$
   - adiabatic: $Q = 0 \implies PV^\gamma = \text{constant}$
     - $\gamma = 5/3 = 1.67$ for monatomic, $7/5 = 1.4$ for diatomic, 1.33 for polyatomic gas
1. Definitions: [T4]
   - **macrostate** specified by macroscopic variables (e.g., 3 of $P$, $V$, $N$, or $T$ for ideal gas)
   - **microstate** specified by quantum state of every molecule
   - **multiplicity** $\Omega$ is number of microstates consistent with a macrostate (e.g., same $U$, $N$)
   - macropartition table:
     - know how to construct one given $\Omega(U, N)$
     - using $U = U_A + U_B = \text{constant}; \Omega_{AB} = \Omega_A \times \Omega_B$
     - predicting the most probable macropartition
   - fundamental assumption: each accessible microstate is equally probable
     $\Rightarrow$ relative probabilities of macropartitions equals ratio of (total) multiplicities

2. Einstein Solid with oscillator energy $\varepsilon = \hbar \omega$ [T4]
   - $\Omega(N, U) = \frac{(3N + U/\varepsilon - 1)!}{(3N - 1)! (U/\varepsilon)!} \quad U = \sum_i n_i \varepsilon \quad U = 3Nk_B T$

3. Temperature and Entropy $S = k_b \ln \Omega$ [T5,T6]
   - $S_{AB} = S_A + S_B$
   - given $S$, find multiplicity $\Omega = e^{S/k_b}$
   - $dS/dU = 1/T$ defines temperature
   - Boltzmann factor: $\Pr(E) = \frac{1}{Z} e^{-E/k_B T} = e^{-E/k_B T} / \sum_{\text{all states}} e^{E_i/k_B T}$
   - Movie test for reversible vs. irreversible processes
   - explain why heat spontaneously flows from hot to cold bodies

   - $\Pr(\text{speed within } dv \text{ centered on } v) = \frac{4}{\pi v_p^2} \left( \frac{v}{v_p} \right)^2 e^{-(v/v_p)^2}$ where $v_p \equiv (2k_B T/m)^{1/2}$
   - Average energy: $E_{\text{avg}} = \sum E_n \left( \frac{e^{-E_n/k_B T}}{Z} \right) = \frac{\sum E_n e^{-E_n/k_B T}}{\sum e^{-E_n/k_B T}}$

5. Calculating Entropy Changes [T8]
   - $\Delta S = \int dQ/T$ (N constant, quasistatic volume change)
   - $\Delta S = 0$ for (quasistatic) adiabatic volume changes
   - isothermal $\Rightarrow \Delta S = Q/T$ ($Q$ given or find from $Q = -W$)
   - phase change: $\Delta S = \pm mL/T$ with $L$ the latent heat
   - If no work, $dQ = dU = mc dT \Rightarrow \Delta S = mc \ln(T_f/T_i)$
   - Non-quasistatic processes $\Rightarrow$ find $\Delta S$ from replacement process