What you should know about . . .

Waves

1. Basic wave properties [Q1]
   - Superposition principle holds if wave equation is linear
   - \( f = \omega/2\pi, \lambda = 2\pi/k \), wave speed \( v = f\lambda = \omega/k \)
   - What does wave velocity depend on? e.g., string: tension and mass per unit length
   - What is resonance?
   - Finding normal modes (as in homework): boundary conditions, counting wavelengths
   - Fourier series: a periodic function can be uniquely decomposed into a sum of sines and cosines

2. Interference phenomena [Q2,Q4]
   - Diffraction through or around object (slit, eyeball, etc.) with size \( a \)
     - total diffraction if \( \lambda > a \); no diffraction if \( \lambda \ll a \)
     - Construction using path difference to derive \( \sin \theta = \lambda/a \) formula for first minimum.
     - When can objects be resolved? (qualitative and quantitative)
   - Applying two-slit interference formula
     - Construction using path difference to derive formula for “bright spots”: \( d\sin \theta = n\lambda \)
   - When is the small-angle approximation (\( \sin \theta \approx \tan \theta \approx \theta \)) valid?

What you should know about . . .

Particlelike Properties of Waves and Wavelike Properties of Particles

1. Photon model [Q3]
   - photon energy/momentum related to frequency and wavelength
   - Basic relations for photons: \( E = hf = hc/\lambda; E = pc \)
   - Basic photon calculations (photons per second emitted, absorbed, visible photon wavelengths, etc.)

2. Photoelectric effect [Q3]
   - predictions of wave vs. particle models [also Q5,Q6]
   - analysis of experiment: \( K = hf - W = hc/\lambda - W \)
   - What is a work function?

3. de Broglie wavelength: what does it mean? [Q4,Q5]
   - \( p = h/\lambda \) and \( E = hf \) (\( E \neq hc/\lambda \) for massive particles!). Nonrelativistic free particle: \( E = p^2/2m \)
   - relationship between kinetic energy and wavelength (qualitative and quantitative)
     - nonrelativistic: \( \lambda = h/\sqrt{2Km} = hc/\sqrt{2Kmc^2} \). Relativistic version: \( \lambda = hc/\sqrt{K(K + 2mc^2)} \)
   - diffraction and interference of particles
     - occurs when deBroglie \( \lambda \) comparable to slit size or other relevant size.
     - applications: electron interference experiments, resolving particles (\( \lambda \lesssim a \))
What you should know about . . .

Quantum Mechanics

1. Probability interpretation of wave function [Q6]
   - Calculating the probability to find a particle in some region of space (given a wave function)
   - Possible results of successive measurements of an observable (e.g., energy or position): “collapse”

2. The probability rule and application to spin wave functions [Q6]
   - If you have quanta in a state $|\psi\rangle$ and measure an observable $O$ of the quanta which has possible eigenvalues $O_n$, $n = 1, 2, 3, \ldots$, then the probability that your measurement yields the specific value $O_n$ for the observable is
     $$Pr(O_n) = |\langle O_n | \psi \rangle|^2,$$
   with $|O_n\rangle$ being the eigenstate of the observable $O$ with eigenvalue $O_n$.

3. Time Dependence of a stationary state [$\propto e^{-iE_n t/\hbar}$] [Q6]
   - When is a solution a “stationary state”? [Definite energy $\implies$ energy eigenfunctions!]
   - What happens if you combine two stationary states?

4. Quanton in a Box [Q7]
   - derivation of energy eigenvalues $E_n = \hbar^2 n^2 / 8mL^2$ and eigenfunctions $\psi_E(x) = A \sin(n\pi x/L)$ in box

5. Bohr model assumptions $\implies$ energy levels $E_n = -ke^2 / 2a_0n^2$ and radii $r_n = n^2a_0$ in hydrogen [Q7]
   - Bohr radius $a_0 = \hbar^2 / 4\pi^2mke^2 \approx 0.053\text{ nm} \implies E_n = -(13.6\text{ eV})/n^2$. Results for hydrogen-like atoms.

6. Photon spectral lines [Q8]
   - Relationship to energy levels (differences!)
   - Infinite well vs. oscillator vs. hydrogen atom (energy levels and spectra)
   - Pauli exclusion principle and spin; fermions vs. bosons

7. Atoms [Q9]: Selection rules (e.g., $\Delta l = \pm 1$), predicting ground state configuration, periodic table of elements

What you should know about . . .

Energy Eigenfunctions [Q10, Q11]

1. Schrödinger eq: $-\frac{\hbar^2}{2m} \frac{d^2\psi_E(x)}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$. Is a given function a possible energy eigenfunction?

2. $\psi_E(x)$ is wavelike (curving towards the axis) in a classically allowed region ($V(x) < E$)

3. The local curvature (2nd derivative!) is related to the local wavelength which is related to the local kinetic energy $K(x) = E - V(x)$ (see equations on previous page and $K = \hbar^2 / 2m\lambda^2$).

4. $\psi_E(x)$ is exponential-like (curving away from the axis) in a classically forbidden region ($V(x) > E$)

5. The wavefunction must decay to zero at $|x| \to \infty$ (normalizability!).

6. This causes quantization of energy: Only for specific values of $E$ does the wavefunction curve in the classically forbidden region in exactly the way needed for the wavefunction to approach zero at $|x| \to \infty$.

7. The quanta spend more time in regions where they move slowly (small $K(x)$) – wave function has larger amplitude there.

8. Tunneling: the ability of a wavefunction to leak through a finite barrier due to its exponential tail in the classically forbidden region.