Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work. Simplify numbers (e.g., write absolute values in terms of real numbers only, reduce answers with phases to expressions using sines and cosines) and normalize vectors where necessary.

1) Math (a) Normalize each vector: (i) \( \frac{1}{\sqrt{2}} |+\rangle - i |\rangle \), (ii) \( \frac{i}{\sqrt{3}} |+\rangle - 2 |\rangle \).

(b) \( M = 2 \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \). Find the eigenvalues and eigenstates of \( M \). Show that the eigenstates are complete or that they are not complete.

(c) Differentiate the following functions (i.e., compute \( \frac{df}{dx} \)). (i) \( f(x) = \frac{1}{1+2x^2} \), (ii) \( f(x) = \cos^2(5x) \), (iii) \( f(x) = \ln[\sin(2x)] \).

(d) Integrate the following functions (i.e., compute the indefinite integral, \( \int dx f(x) \)). (i) \( f(x) = \frac{1}{1+x^2} \), (ii) \( f(x) = \exp[-i x] \), (iii) \( f(x) = \sin(\omega x + \phi) \).

2) A spin-1/2 particle is governed by the hamiltonian, \( \hat{H} = \frac{\omega}{\sqrt{2}} (S_x - S_y) \). At time \( t = 0 \) a beam is sent through a Stern-Gerlach device oriented in the z direction and the ‘lower’ beam is selected.

(a) At time \( t = t_1 \), \( S_x \) is measured. What are the possible results and their respective probabilities?

(b) Compute \( \langle S \rangle \) for \( 0 \leq t \leq t_1 \).

(c) If the measurement at time \( t_1 \) produces \( +\frac{h}{2} \) and a second measurement, this time of \( S_z \), is made at time \( t = 2t_1 \), what are the possible results and their respective probabilities?

3) A spin-1 particle is governed by \( \hat{H} = \omega S_x \).

(a) Compute the eigenvalues and eigenstates of \( S_x \). It is not enough to produce the results from memory; show the calculations.

(b) At time \( t = 0 \), we measure \( S_z \) and get zero. At time \( t = t_1 \), we measure \( S_z \) again. What are the possible results and their respective probabilities?

(c) What is the expectation value of \( S_y \), as a function of time?