Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work. Simplify numbers (e.g., write absolute values in terms of real numbers only, reduce answers with phases to expressions using sines and cosines) and normalize vectors where necessary.

1) **Particle on a Ring (radius R) in a Magnetic Field**. $H = \frac{L^2}{2I} - \frac{L_z}{I}$. The initial state is, $|\psi(0)\rangle = N[\sin(2\phi) + \exp(i\phi)]$. **All measurements below are made at a later time** $t$. All integrals in this problem should be evaluated.

(a) What are the possible results for a measurement of $L_z$ and their probabilities?
(b) What is $\langle H \rangle$?
(c) What is $\langle Y \rangle$?
(d) If a measurement were made at $t = 0$, and it was found that $L_z > 0$, what is $\langle X \rangle$ at the later time, $t$?

2) **Particle on a Sphere in a Magnetic Field**. $H = \frac{L^2}{2I} + \frac{3L_z^2}{2I}$. The initial state is: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}|0,0\rangle + \frac{i}{\sqrt{2}}|1,0\rangle - \frac{1}{\sqrt{2}}|1,-1\rangle$. **All measurements below are made at a later time** $t$.

(a) What are the eigenstates and eigenvalues of $H$? What is the ground state(s)? What is the first excited state(s)?
(b) What is $\langle L_z \rangle$? What is $\langle H \rangle$?
(c) Assume the fixed radius is $R$. What are $\langle X \rangle$ and $\langle Z \rangle$?

3) **Hydrogen-like Atom**. $H = \frac{P^2}{2m_e} - \frac{Z\alpha}{r}$. **All measurements below are made at a later time** $t$. At $t = 0$ the electron is bound to a Helium-3 nucleus ($Z = 2$) in the $Z = 2$ ground state. An unknown interaction violating several current laws of physics (i.e., conservation of momentum, conservation of energy, conservation of angular momentum) takes place at $t = 0$ and one of the protons becomes a neutron, so that $Z = 1$ after $t = 0$. So, we start with a Helium isotope and end with tritium, a hydrogen isotope.

(a) What are $\langle H \rangle$ and $\langle L^2 \rangle$?
(b) What is the probability of finding the tritium atom in its ground state? What is the probability of finding it in an $n = 2$ state?
(c) What is \( \langle r \rangle \)?

Extra Credit) At \( t = 0 \) the electron, in a hydrogen ground state, is found to satisfy \( X > 0 \). The wave function collapses. At time \( t \), measurements are made. Compute: (i) \( \langle L_z \rangle \), (ii) \( \langle L^2 \rangle \), (iii) \( \langle H \rangle \), (iv) \( P(n = 1) \).