Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Two particles (same mass but not identical) are confined in a one-dimensional square well, so that $-L/2 < x_i < L/2$. (a) What are the energy eigenstates in position representation and their eigenvalues? (b) A perturbation is added:

$$H' = g \delta(x_1)\delta(x_2).$$

Use perturbation theory to compute the first-order shift in the energies of all states for which $n_i = 1, 2, 3$.

2) A hydrogen atom, $H_0 = \frac{p^2}{2m_e} - \frac{\alpha}{r}$, is subject to a simplified (and totally unrealistic) spin-orbit interaction:

$$H_{SO} = \eta \mathbf{L} \cdot \mathbf{S},$$

where $\eta$ is a constant, $\mathbf{S}$ is the electron spin, and you can ignore the proton spin. (a) A complete set of $H_0$ eigenstates is $|n, l, m_l, m_s\rangle$, with eigenvalues $-\frac{Ryd}{n^2}$. What are the eigenstates and eigenvalues for $H_0 + H_{SO}$? (b) An electric field is turned on, adding the perturbation:

$$H' = -e\mathbf{E}z.$$ 

Compute the first-order energy shifts for all $n = 2$ states.

3) Two particles, the first spin-1 and the second spin-1/2, are confined to move on the surface of a sphere. In addition to their kinetic energies, they experience spin-orbit interactions, so that:

$$H = \frac{L_1^2}{2I} + \frac{L_2^2}{2I} + \frac{L_1 \cdot S_2}{I} + \frac{L_2 \cdot S_1}{I}.$$ 

(a) What are the energy eigenstates and eigenvalues? Clearly define any angular momentum operators you use to label the eigenstates and specify the range of values allowed for any variables (e.g., $l_1 = 0, 1, 2, ...$). (b) Find the two lowest energy “shells.” What are their energies? List all of the states in each shell. (c) A complete set of measurements is made at $t = 0$ and it is found that $l_1 = 1, m_{l_1} = 1, m_{s_1} = 0$ and $l_2 = 0, m_{l_2} = 0, m_{s_2} = -1/2$. What is the probability of finding these same results at a later time, $t$?