Angular Momentum  \( J^+ = J_x + iJ_y \)

\[ J^2|j, m\rangle = j(j+1)|j, m\rangle; \quad J_z|j, m\rangle = m|j, m\rangle \]

\[ J^+|j, m\rangle = \sqrt{j(j+1)-m(m+1)}|j, m+1\rangle \]

Harmonic Oscillator  \( H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = (\alpha + \frac{1}{2})\omega \)

\[ a = \sqrt{\frac{m\omega}{2}} \left( x + \frac{i\hbar}{m\omega} \right); \quad a^+ = \sqrt{\frac{m\omega}{2}} \left( x - \frac{i\hbar}{m\omega} \right) \]

\[ [a, a^+] = 1 \Rightarrow H|n\rangle = (\alpha + \frac{1}{2})\omega |n\rangle \]

\[ a|n\rangle = \sqrt{n} |n-1\rangle; \quad a^+|n\rangle = \sqrt{n+1} |n+1\rangle \]

Hydrogen-like atoms  \( H = \frac{p^2}{2m} - \frac{Z\alpha}{r} \)

\[ \alpha = \frac{\hbar}{2mc} \quad E_n = \frac{1}{n^2} \left( -\frac{1}{2} Z^2 \alpha^2 mc^2 \right) \]

\[ R_{10}(r) = 2 \left( \frac{Z}{a_0} \right)^{\frac{3}{2}} e^{-Zr/\alpha_0} \quad y_0 = \sqrt{\frac{1}{4\pi}} \]

\[ R_{20}(r) = 2 \left( \frac{Z}{a_0} \right)^{\frac{3}{2}} \left[ -\frac{Zr}{2a_0} \right] e^{-Zr/\alpha_0} \quad y_0 = \sqrt{\frac{3}{2\pi}} \cos \theta \]

\[ R_{21}(r) = \frac{1}{\sqrt{3}} \left( \frac{Z}{a_0} \right)^{\frac{3}{2}} \frac{Zr}{\alpha_0} e^{-Zr/\alpha_0} \quad y_1^\pm = \pm \frac{\sqrt{3}}{8\pi} \sin \theta \]

\[ \langle F | H(t') | i \rangle e^{-i\omega t'} |^2 \]
36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over every coefficient, e.g., for \(-8/15\) read \(-\sqrt{8/15}\).

![Diagram of Clebsch-Gordan coefficients and spherical harmonics with formulas and coefficients, including notation for \(d_{m,0}\), \(d_{1/2,0}\), and \(d_{1/2,1}\) coefficients.](image)

\[ Y^{m}_{\ell} = (-1)^{\ell-m} Y^{m}_{\ell} \]

\[ d_{m,0} = \sqrt{2} \sin \theta \]

\[ d_{1/2,0} = \cos \theta \]

\[ d_{1/2,1} = \sin \theta \]

\[ d_{1/2,1} = \cos \theta \]

\[ d_{1/2,1} = -\sin \theta \]

\[ d_{1/2,1} = -\cos \theta \]

**Figure 36.1:** The sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).