HONORS PHYSICS 5501H     Spring 2016     Exam #3

Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) [Atomic Slowing of Hydrogen] This is a “back of the envelope” type of calculation in which you actually need to obtain numbers, precise to only one digit. All constants and formulae you need are given in the notes. Hydrogen atoms are emitted from an “oven” at room temperature ($kT \approx 1/40$ eV). You can ignore the important subtleties associated with Doppler shifts in this problem. (a) What should the frequency, $\nu$, of the laser be to slow the hydrogen atoms? (b) What is the average change in momentum of a hydrogen atom for each photon absorbed? (c) How many photons must be absorbed to stop a hydrogen atom? (d) If the intensity of the laser is adjusted so that it takes about 10 times as long for a new photon to be absorbed as it takes for the atom to decay back to the ground state after absorbing one photon, how far will the atoms travel before being stopped?

2) [Continuous Media and LCAO] A string of $N$ attractive $\delta$-function potentials, with alternating strengths, are equally distributed on a very large circle of circumference $Na$. So, $N/2$ of the potentials are $-g_1\delta(x)$ and between these are $N/2$ potentials $-g_2\delta(x)$. The whole potential can be written as:

$$V(x) = \sum_{n=1}^{N/2}(-g_1\delta(x - 2na)) + \sum_{n=1}^{N/2}(-g_2\delta(x - 2na - a))$$

Use the LCAO approximation, with different bound states for the different strength well, and the nearest-neighbor approximation to derive dispersion relation(s). Note that the nearest neighbor for each well is a well of different strength. Specify all constants in $E(k)$ in terms of $g_1, g_2, m$ and $a$.

[ALTERNATE PROBLEM: For 10 points less, solve for $g_1 = g_2$.]

3) [Quantum Computing] (a) Verify the Deutsch algorithm for the function: $f : 0 \rightarrow 1, 1 \rightarrow 1$. First design a unitary transformation, $U_f$, that implements this function. Initiate with $|01\rangle$, apply Hadamard gates $H_x H_y$ to this state and then apply the function, $U_f$. Finally apply $H_x$ to the resultant state and explicitly show the final state. Verify that this procedure accomplishes what the Deutsch algorithm is designed for.

(b) Carol has a secret:
\[ |\psi\rangle_{\text{secret}} = \frac{1}{\sqrt{3}} |0\rangle + i\sqrt{\frac{2}{3}} |1\rangle, \]

which she wants to get to Bob. She gives the 1st qubit of a key, \(|\beta_{00}\rangle\), to Alice and the 2nd to Bob. She also gives the secret to Alice. Alice first performs an operation that maps Bell states to computational states (\(|\beta_{00}\rangle \rightarrow |00\rangle\), etc.). She then measures her qubits to get 0 for the key and 0 for the secret. When told these results, Bob does nothing to his qubit. Show that Bob's qubit is left in the secret state.