Solution to Set 2

2-29

Choose the direction pointing downwards along the slope is positive. Then the net force acting on the automobile along the slope is

$$f = mg\sin\theta - \mu_k mg\cos\theta$$

the equation of motion is

$$m\frac{dv}{dt} = mg\sin\theta - \mu_k mg\cos\theta \ .$$

So

$$\frac{dv}{dt} = g\sin\theta - \mu_k g\cos\theta$$
$$v\frac{dv}{dt} = (g\sin\theta - \mu_k g\cos\theta)v$$
$$\frac{1}{2}\frac{dv^2}{dt} = (g\sin\theta - \mu_k g\cos\theta)v$$
$$dv^2 = 2(g\sin\theta - \mu_k g\cos\theta)v\,dt$$
$$dv^2 = 2(g\sin\theta - \mu_k g\cos\theta)dx$$

$$\int_{v_0}^{v} dv^2 = \int_{x_0}^{x} 2(g\sin\theta - \mu_k g\cos\theta)dx$$
$$v^2 - v_0^2 = 2(g\sin\theta - \mu_k g\cos\theta)(x - x_0)$$
$$v_0 = \sqrt{v^2 - 2(g\sin\theta - \mu_k g\cos\theta)(x - x_0)}$$

Here $x - x_0 = 30m, v = 0, \mu_k = 0.45$ and $\tan\theta = 0.08$,so $\sin\theta = 0.0797, \cos\theta = 0.9968$

$$v_0 = 14.727m/s = 32.95MPH$$

1 mile = 1.609km

2-37

The force acting on the particle is

$$F(x) = ma = m\frac{dv}{dt}.$$

And the speed is $v = \alpha / x$, so

$$\begin{aligned} F(x) &= m\frac{dv}{dt} \\ &= m\frac{d}{dt}\left(\frac{\alpha}{x}\right) \\ &= -m\frac{\alpha}{x^2}\frac{dx}{dt} \\ &= -m\frac{\alpha}{x^2}v \\ &= -m\frac{\alpha}{x^2}\frac{\alpha}{x} \\ &= -m\frac{\alpha^2}{x^3} \end{aligned}$$

2-39

Assume the mass of the boat is m . The equation of motion is

$$m\frac{dv}{dt}=-\alpha\exp(\beta v)$$

$$\exp(-\beta v)dv=-\frac{\alpha dt}{m}$$

$$\int_{v_0}^{v}\exp(-\beta v)dv=\int_0^t-\frac{\alpha dt}{m}$$

$$-\frac{1}{\beta}\left(\exp(-\beta v)-\exp(-\beta v_0)\right)=-\frac{\alpha t}{m}$$

so $$v=-\frac{1}{\beta}\ln\left(\frac{\alpha\beta t}{m}+\exp(-\beta v_0)\right).$$

Let $v=0$, then we can get

$$t=\frac{m}{\alpha\beta}\left(1-\exp(-\beta v_0)\right).$$

Since $$\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx},$$

$$\exp(-\beta v)vdv=-\frac{\alpha}{m}dx$$

then $$\int_{v_0}^{0}\exp(-\beta v)vdv=\int_0^x-\frac{\alpha}{m}dx.$$

$$-\frac{1}{\beta^2}+\frac{\exp(-\beta v_0)(1+\beta v_0)}{\beta^2}=-\frac{\alpha}{m}x$$

Therefore, the distance is $x=\frac{\alpha}{m}\frac{1-\exp(-\beta v_0)(1+\beta v_0)}{\beta^2}$

2-56

Assume the mass of the launcher at time t is m and the positive direction is pointing upwards.

Then the equations of motion are

for launcher: $$F-(m+dm)g^{'}=m\frac{dv_m}{dt}$$

for the emitted mass: $$-(F-g^{'}dm)=-dm\frac{dv}{dt}$$

So $$m\frac{dv_m}{dt}+mg^{'}=dm\frac{dv}{dt}$$

Here $$\frac{dv_m}{dt}=0, dv=-u$$

Then
$$mg^{'}=-u\frac{dm}{dt}$$
$$dt=-\frac{u}{g^{'}}\frac{dm}{m}$$

$$\int_0^t dt=\int_{m_0}^{m}-\frac{u}{g^{'}}\frac{dm}{m}$$
$$t=-\frac{u}{g^{'}}\ln\frac{m}{m_0}$$
$$=-\frac{2000m/s}{9.8m/s^2/6}\ln\frac{0.8m_0}{m_0}$$
$$\approx 273.24s$$

3-4

Since

$$x(t) = A\sin(\omega_0 t - \delta)\,,$$

the kinetic energy and the potential energy are

$$T = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t - \delta)$$

and $$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2(\omega_0 t - \delta) = \frac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t - \delta)$$ respectively.

Then

$$\langle T \rangle_t = \frac{1}{T}\int_0^T \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t - \delta)dt = \frac{1}{4}m\omega_0^2 A^2$$

$$\langle U \rangle_t = \frac{1}{T}\int_0^T \frac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t - \delta)dt = \frac{1}{4}m\omega_0^2 A^2\,.$$

$$\langle T \rangle_x = \frac{1}{2A}\int_{-A}^A \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t - \delta)dx = \frac{1}{3}m\omega_0^2 A^2$$

$$\langle U \rangle_x = \frac{1}{2A}\int_{-A}^A \frac{1}{2}m\omega_0^2 A^2 \sin^2(\omega_0 t - \delta)dx = \frac{1}{6}m\omega_0^2 A^2$$

3-5

For a simple harmonic oscillator,

$$x = A\sin(\omega_0 t - \delta)$$

so

$$\frac{dx}{dt} = A\omega_0 \cos(\omega_0 t - \delta) = \omega_0 \sqrt{A^2 - x^2}$$

$$\frac{dt}{T} = \frac{dx}{\omega_0 \sqrt{A^2 - x^2}}\frac{1}{T} = \frac{dx}{\omega_0 \sqrt{A^2 - x^2}}\frac{\omega_0}{2\pi} = \frac{dx}{2\pi\sqrt{A^2 - x^2}}$$

$$\frac{\Delta t}{T} = \int_x^{x+\Delta x} \frac{dx}{2\pi\sqrt{A^2 - x^2}}$$

$$= \frac{1}{2\pi}\left(Arc\sin\frac{x+\Delta x}{A} - Arc\sin\frac{x}{A} \right)$$

$$or\ \frac{1}{2\pi}\left(Arc\tan\frac{x+\Delta x}{\sqrt{A^2 - (x+\Delta x)^2}} - Arc\tan\frac{x}{\sqrt{A^2 - x^2}} \right)$$