

Write your name on the test booklet. Do NOT simply write an answer. Clearly show the steps in each calculation. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1a) Find all solutions to $\sin(z) = 4$ and show them on a plot in the complex plane.

1b) Find all solutions to $z^i = 2$ and show them on a plot in the complex plane.

2a) Find the real and imaginary parts u and v of $f = \sqrt{z}$. Determine whether and where they obey the Cauchy-Riemann relations and compute df/dz . Display any singularities (including branch cuts) on a plot in the complex plane.

2b) Find the Taylor or Laurent expansion of $f = \frac{z^2}{1-z^2}$ around $z = 0$ and $z = -1$. What is the radius of convergence in each case?

2c) Find the Taylor or Laurent expansion of $\ln(z)/(z^2 + 1)$ about $z = i$. What is the radius of convergence?

3a) Evaluate $\int_0^{2\pi} \frac{1}{2+\sin(\theta)+\cos(\theta)} d\theta$.

3b) Evaluate $\int_0^\infty \frac{x^{1/2}}{x^2+1} dx$.

3c) Evaluate $\int_{-\infty}^\infty \frac{\cos(x)}{x^4+x^2+1} dx$.

4) A damped, driven harmonic oscillator satisfies the equation:

$$m \frac{d^2x}{dt^2} + r \frac{dx}{dt} + kx = F(t)$$

The initial position and velocity are zero, at which time a force is applied so that $F(t) = F_0\theta(t)\theta(\tau - t)$. In other words it is turned on at $t = 0$ and off at $t = \tau$. Assume $r/2m > \sqrt{k/m}$. Find the position, $x(t)$.

5) The Stark effect describes the energy shift of atomic levels in an electric field. The equation is:

$$x y'' + y' + (k - E/4 x^2 + U/2 x - m^2/(4x)) y = 0$$

where the term $E/4x^2$ is the electric field perturbation. Obtain a power series solution for y and obtain explicit expressions for the first four nonzero terms. How many terms are needed before any effect of the electric field is found?