

Write your name on the test booklet. Do NOT simply write an answer. Clearly show the steps in each calculation. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

- 1a) Find the full Fourier series for $f(x) = x$ over the range $0 \leq x \leq 1$.
 1b) A spring-and-dashpot system satisfies the equation:

$$\frac{d^2x}{dt^2} + \alpha \frac{dx}{dt} + k^2 x = f(t)$$

$f(t) = at$ for $0 \leq t \leq T/2$ and $f(t) = a(T - t)$ for $T/2 \leq t \leq T$. Find $x(t)$ as a Fourier series.

2a) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} \delta(x^2 + x - 6)$.

2b) A line of charge with uniform line charge density λ lies along the z-axis. Find the volume charge density in (i) cylindrical coordinates and (ii) spherical coordinates.

- 3a) Sum the series:

$$\sum_{p=0}^{\infty} (-1)^p \frac{2p+1}{x^2 + (2p+1)^2}$$

by taking the Fourier transform of each term, summing the series in the transform space, and then transforming back.

- 3b) A spring-and-dashpot system satisfies the equation:

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + \omega_0^2 x = f(t)$$

with $\omega_0 > \alpha$ and $f(t) = e^{-\alpha t} \sin(\Omega t)$ for $t > 0$. Find $x(t)$ for $t > 0$.