Supercontinuum Generation in Sapphire: A Measurement of Intensity

Courtney Jarman¹, Douglass Schumacher² (Advisor), Camelia Modoran², Nick Harmon², Zachary Yoscovits²

[1] The University of Michigan, Ann Arbor, MI 48109
[2] The Ohio State University, Columbus, OH 43210

We have measured the intensity of a single plasma filament of an 800 nm short pulse laser while supercontinuum generation (CG) occurs in a 3 mm thick sapphire crystal (see figure above)¹. In order to find this we first needed to determine the diameter of the filament that exists within the crystal. The resulting spot size of around 20 microns matches well with what others have found in the past.

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I. Introduction

The generation of many colors from a nearly monochromatic light source when passed through a transparent medium is not an uncommon phenomenon. However, today the process of supercontinuum generation is still not perfectly understood, and there is a great deal to be learned about its basic elements.

The plasma that forms during this CG and the properties of the filament(s) that exist within the medium have never been accurately characterized. The overall purpose of our experiment this summer was to determine the size and shape of the central filament of the plasma during CG in a sapphire crystal, which would eventually allow us to measure its intensity. The results of our analysis will also help us to start an electro-sampling experiment to measure the free electron density of the plasma that is created during this process.

A detailed understanding of the properties of the supercontinuum is crucial for assessing the possibilities of using this broadband emission for spectroscopic, metrological, biomedical, and pulse-compression purposes.

One application of CG is its use in optical coherence tomography, where cell level abnormalities can be detected in skin and soft tissues. CG is also being used to create the world’s most accurate clocks and time standards. Physicists also hope to one day use CG to create “tunable” lasers that are capable of outputting a broad range of frequencies. Other applications include signal processing, three-dimensional imaging, ranging, and atmospheric remote sensing.

II. Background

Before going into detail about our experiment, the basics of CG must be explained. The process is caused by the nonlinear effects of high power ultra short (laser) light pulses passing through a centrosymmetric transparent medium. When the E-field of the light becomes comparable in strength to the material’s atomic field, the higher order nonlinear terms of the polarization expansion have certain effects.

\[ P = \epsilon_0 \chi^{(1)} E + \epsilon_0 \chi^{(2)} EE + \epsilon_0 \chi^{(3)} EEE + \ldots \]  

As each nonlinear term of this equation comes into play, new physical processes can be observed in the sample. In this case we are only concerned with centrosymmetric materials, for which all of the even order terms drop out. Therefore we can ignore the second order nonlinear term. The third order term in this equation leads to the processes of self-focusing and self-phase modulation (SPM), which are the bases of the formation of CG.

Both self-focusing and SPM can be explained using the basic nonlinear

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CG equation, which can be derived from equation 1 (see appendix A):

\[ n = n_0 + n_2 I(r,t) \]  

(2)

where \( n_0 \) and \( n_2 \) are constants.

As you can see from the above equation, the nonlinear index of refraction is dependent upon the intensity of the laser beam, which in turn has both spatial and time dependence.

Self-focusing of the laser arises from the Gaussian-shaped spatial variation of intensity across the beam. The beam tends to be more intense in the middle than on its edges, so according to equation 2, the middle of the beam will have a higher index of refraction than the edges, and will therefore travel more slowly. The effect is that the beam starts to focus.

It should be noted that in order for self-focusing to start in the medium, two things must occur. First of all, the beam must be given a “jump start” in its focusing before entering the medium, say by means of a lens. In our case we used a 100 mm focal length lens to focus into the crystal. Secondly, the power of the laser must exceed a certain threshold power characteristic of the material being used and the wavelength of the laser.\(^7\)

\[ P_{th} \approx 4 \frac{\lambda_0^2}{8\pi n_0 n_2} \]  

(3)

Unless this critical power is reached, there can be no self-focusing, and no CG will occur. One strong connection between CG and self-focusing is that they are both triggered by the same \( P_{th} \).

When \( P_{th} \) is reached, a plasma is formed in the material due to multiphoton ionization, which defocuses the beam and prevents it from merging to a single point. How much power is needed for multiphoton ionization depends on the band gap of the medium.\(^8\) The free electrons reduce the index of refraction of the material according to the equation:\(^7\)

\[ \Delta n = -\frac{2\pi e^2 N_e}{n_0 m_e (\omega_0^2 + \nu^2)} \]  

(4)

where \( N_e \) is the electron density, \( \omega_0 \) is the central frequency, \( \nu \) is the electron collision frequency, and \( m_e \) is the mass of an electron.\(^7\) Self-focusing and the defocusing due to the free electrons come to an equilibrium when \( N_e \) reaches \( 10^{17} \text{ electrons/cm}^3 \) because \( \Delta n \) cancels out \( n_2 I(r,t) \). The beam then becomes collimated and its diameter stays about the same size through the rest of the medium.

\[ \Delta n = \frac{\sum_{i} \Delta n_i}{\sum_{i} n_i} \]  

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At the threshold power there is an increase in intensity because the beam’s profile gets smaller through self-focusing. This helps to enhance self-phase modulation, which is another process that contributes to supercontinuum generation. This SPM is again based on the intensity dependence of the index of refraction, but is created due to the time dependence of the intensity (see appendix B):

\[
\frac{\partial \omega}{\partial t} = \omega_0 - n_2 \frac{\partial}{\partial z} \frac{\partial I}{\partial t}
\]  \hspace{1cm} (5)

Looking at equation 5, it can be seen that an increase in intensity will cause a red shift in the laser while a decrease in intensity will cause a blue shift in the beam.

This time variation in intensity is what creates a variety of frequencies and colors from a nearly monochromatic light source.

III. The Experiment

A. Spot Size

In order to find the intensity of a single filament in the sapphire, we first needed to know the diameter of the filament as it comes out of the back of the crystal (after self-focusing). There have been attempts by others to measure this spot size in the past, but there is currently no widely accepted value. For example, in 2000 a group of researchers reported that the beam “shrinks to a universal diameter of 20-25 micrometers in 4 mm propagation distance”\(^8\). We used sapphire as our transparent medium because it is known to produce an excellent CG and it is not easily damaged by the laser.

1. Methods

To create this CG we used an 800 nm Ti:sapphire laser with a bandwidth of about 10 nm. Our laser has a power of about 800 mW, a pulse width of 100 fs and a repetition rate of 1 KHz. It uses chirped pulse amplification to maximize its power while minimizing its pulse width. Our crystal was 3 mm thick.

Our first task was to set up the optical instruments in such a way as to align the beam directly into a CCD.

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camera, and to set up a “microscope” so that the final image was magnified. We put 2 apertures in the beginning of the beam’s path so as to keep it moving in a straight line, and then reflected the beam off of 2 adjustable mirrors so that we could easily maneuver it into the camera.

Next, we set up a 100 mm focal length lens in order to focus the beam directly into the sapphire with the focal point somewhere in the middle of the sample. We also used a 75 mm focal length lens to magnify the image onto the CCD camera.

Image distance $\approx 1.5m$
Magnification $= -I/O \approx -1.5/0.075 = -20$

We estimated that our object distance was about the same as the focal length of the magnifying lens, to within a couple centimeters. It was not necessary to get an exact measurement at first because we set up a translation table under the lens so that we could move it back and forth until the object was in focus. We also took the lens out of the front of the CCD camera so that the incoming beam could not focus on the face of the camera and damage it.

We added a telescope to the setup in order to correct some astigmatism in the beam. The telescope allowed us to have a larger but still collimated beam that we could then clip off into a smaller circular shape before aligning it into the crystal. We used a -100mm and a 200mm focal length lens for the telescope.

Telescope setup:

![Telescope setup diagram]

We then sent the beam through a variable attenuator with a small circular opening so that we could cut off the edges of the beam. The attenuator also allowed us to reduce or increase the power of the beam with an adjustable polarizer. To account for diffraction effects we put another aperture directly in front of the 100 mm lens and closed it just enough to cut off the diffraction rings of the beam. We also set up neutral filters before the sapphire so that we
could easily adjust the power going into the crystal, and again before the camera so that we did not saturate or damage it.

Calibrating the Image

We connected a frame grabber to the CCD camera in order to display the image on a computer monitor. We then placed wire mesh with a 25.4 micron diameter in the spot where the crystal would be and slightly adjusted the 75mm lens in order to focus the image on the computer screen. We measured the number of pixels across the wire horizontally and vertically and compared it to the actual distance across the wire (25.4 microns) in order to get a ratio. We found that there were 0.5184 microns/pixel vertically and 0.5644 microns/pixel horizontally. We confirmed these measurements by comparing them to the pixel ratio of the camera (492 x 542), and found that the 492 vertical pixels and 542 horizontal pixels of the camera gave it a 1.10 pixel ratio, which is about the same as 0.5644/0.5184.

Imaging the sapphire:

Our last task was to adjust the 75 mm lens until we were imaging the back surface of the crystal. To do this, we used a diamond scribe to scratch a small “X” on the back of the sapphire. After we adjusted the power of the laser and the position of the crystal so as to get a steady CG, we moved the 75 mm lens slightly (to change the image distance) until we got a clear picture of the scratch on the screen. Lastly, we moved the crystal laterally so that we were still imaging the back of the sapphire without hitting the scratch. We captured this final picture with the frame grabber and analyzed the data in Matlab. We took a few good trials and made sure to measure the power of the beam going into the crystal.

2. Data Analysis

Here is a picture we created in Matlab of the “X” scratch on the back of the crystal:

![Image of scratch](image.png)

We also analyzed a frame without any beam whatsoever so that we could subtract out the background light, but the camera was not sensitive enough to pick up any outside light.

We created a program to scale the image to size and to convert pixels to microns on the x and y-axis. We also made a program that takes a 2D slice of the data through the maximum intensity point, and is able to rotate this plane through 180 degrees. This program then graphs the “slice” of data on a 2D plot. It can also find the maximum, minimum and average spot size of the data as well as the angle of the “slicer” plane where these particular maxima and minima occur. Here is an example of the output:
The bottom graph shows the 2D slice where the spot size is at a minimum, and the top contour graph shows where exactly on the data this slice is being taken. The diameter of the spot (“width”) and the theta where the minimum occurs are displayed on top of the graph. We took all spot size measurements at full width, half max.

Our final data shows that the average spot size was about 20.1 microns, which is similar to what others have measured in the past. We also took power measurements of the incoming beam for each spot size measured.

From this graph you can see that after the threshold power was reached (around 0.75 mW) and CG occurred, the spot size of the laser stayed nearly the same to within about 3 microns. The power in this data set was too low to create multifilamentation, so these are all measurements of a single filament. We later increased the power to observe the multifilaments.

With the diameter and power measurements above, we were able to calculate four intensity values:

\[
\begin{align*}
4.81 \cdot 10^{12} \frac{W}{cm^2} \\
5.51 \cdot 10^{12} \frac{W}{cm^2} \\
4.01 \cdot 10^{12} \frac{W}{cm^2} \\
2.41 \cdot 10^{12} \frac{W}{cm^2}
\end{align*}
\]

These are all similar measurements, with an average intensity of \(4.19 \cdot 10^{12} \frac{W}{cm^2}\).

IV. The Next Step
After knowing the diameter of the central filament, we were able to begin our next project. The specific details of the plasma formed in the sample during a supercontinuum generation have never been accurately characterized. Our task was to find out more about the amount of free electrons that are created during the CG process.

In order to take measurements of the free electron density, we have to connect electrodes to the back surface of the sapphire while a laser excites the crystal into a plasma. To do this we will evaporate a conducting metal (aluminum) onto the sample and connect small wires from these electrodes to an ammeter to measure the current.

We want to have a gap between the electrodes that is almost identical in size to the measured diameter of the beam so that we can sample as many free electrons as possible. Before concluding the previous experiment we did not know how far apart to place the evaporated metal because we did not know the actual spot size of the laser. After we had our final data for the diameter of the beam, we created a mask that shielded only a small 20 micron portion of the crystal from the evaporating aluminum. We had many ideas on how to make this mask, but the final decision was to set a thin wire directly on the sapphire while the evaporation was taking place.

We placed a circular wire that was 25.4 microns in diameter across the center of the practice sample. It was assumed that the evaporating metal would create a gap that was a bit smaller than the 25.4 microns, since the wire we used was circular.

After we tested the mask and evaporation experiment on a piece of glass, we used diffraction to find the gap width. We shined a 632.8 nm HeNe laser through the slit and got a diffraction pattern downstream. We then used the diffraction equation \( D \sin \theta = m \lambda \) to find out that our slit width was 20.55 ± 0.50 microns.

The next step is to use the mask to evaporate aluminum electrodes onto the sapphire. After that, the actual electro-sampling experiment may be performed on the crystal.

V. Conclusion

The future technology of supercontinuum generation is very promising. However, at this time more information about the process is needed in order to finish construction on its many applications.

Through my research this summer I not only learned a lot about the CG phenomena, but I was also able to contribute to a better understanding of its details. We observed and recorded consistent patterns in the filaments’ diameter and intensity, and began a new experiment that will help us to characterize the number of free electrons generated in the sapphire during the process.

This experience has helped me discover first hand the procedures of the scientific process. As a student I have gained many valuable research skills that will give me a head start in my long term career.
Appendix A:

(1) \( P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} EE + \varepsilon_0 \chi^{(3)} EEE + \ldots \)

After dropping the second order nonlinear term, the total polarization equation can be expressed as:

\[
P = P^{(1)} + P^{(3)}
\]

\[
P = \varepsilon_0 \chi^{(1)} E + \frac{3}{4} \varepsilon_0 \chi^{(3)} |E|^2 E.
\]

Since the E-field of the laser can be expressed in terms of intensity, we can make a substitution into the above equation:

\[
I = \frac{1}{2} \varepsilon_0 c n_0 |E|^2
\]

\[
P = \varepsilon_0 (\chi^{(1)} + \frac{6}{4 \varepsilon_0 c n_0} \chi^{(3)} I) E
\]

\[
= \varepsilon_0 \chi_{\text{eff}} E
\]

This new effective susceptibility, \( \chi_{\text{eff}} \), is dependent upon intensity, and so the new index of refraction is:

\[
n = \sqrt{\varepsilon} = \sqrt{1 + \chi_{\text{eff}}}
\]

\[
= \sqrt{1 + \chi^{(1)} + \frac{6}{4 \varepsilon_0 c n_0} \chi^{(3)} I}.
\]

This complicated equation can be rearranged to a simpler form:

\[
2
\]

\[
n = n_0 + n_2 I(t)
\]

where \( n_2 = \frac{3 \chi^{(3)}}{4 \varepsilon_0 c n_0^2} \) and is called the nonlinear index of refraction.

Appendix B:

\[
n(I) = n_0 + n_2 I(t)
\]

\[
n = n(t)
\]

If the E-field of the incoming beam can be expressed as:

\[
E \sim E_0 e^{i\phi}
\]

and the phase of the E-field can be expressed as:

\[
\phi = k z - \omega_0 t
\]

\[
k = \frac{\omega_0}{c} = \frac{n(t)}{\nu} \omega_0
\]

\[
\phi = n(t) \frac{\omega_0}{c} z - \omega_0 t
\]

(substitution of \( n(t) \))

\[
\phi = n_0 \frac{\omega_0}{c} z + n_2 \frac{\omega_0}{c} I(t) z - \omega_0
\]

then the frequency of the light can be expressed as the time derivative of the phase:

\[
\omega = -\frac{d\phi}{dt} = \omega_0 - n_2 \frac{\omega_0}{c} z \frac{dI}{dt}
\]