Physics 827: Problem Set 3

Due Wednesday, October 10 by 11:59 PM

Each problem is worth 10 points.

1. Shankar, problem 1.9.2.

2. Shankar, problem 1.9.3.

3. Shankar, problem 1.10.3.

4. Consider the operator $T = CK^2$, where $K = -iD$ and D is the derivative operator in one dimension, as discussed in class (see Shankar, pp. 63-67), and C is a real scalar constant. Assume that T acts on the entire real axis, $-\infty < x < \infty$.

   (a). Show that T is Hermitian.

   (b). Find the eigenvalues and eigenvectors of T. Write the eigenvalues as $-Ck^2$ and the corresponding eigenvectors as $|k\rangle$, so that $T|k\rangle = -Ck^2|t\rangle$. What are the eigenvectors $\langle x|k\rangle$ in the x basis? Choose the normalization $\langle k|k'\rangle = \delta(k - k')$.

5. As discussed in class and in Shankar, the classical Hamiltonian is defined by

$$\mathcal{H} = \sum_{i=1}^{n} p_i \dot{x}_i - \mathcal{L}, \qquad (1)$$

   where $\mathcal{L}$ is a function of the n coordinates x_i and n velocities $\dot{x}_i$, and $p_i = \partial\mathcal{L}/\partial\dot{x}_i$.

   Show that if $\mathcal{L} = T - V$, and $T = \sum_i \sum_j T_{ij}\dot{x}_i\dot{x}_j$, where the T_{ij}'s are constants, then $\mathcal{H} = T + V$. You may assume with no loss of generality that $T_{ij} = T_{ji}$.

6. Shankar, problem 2.7.2 (i).

7. OPTIONAL; NOT TO BE TURNED IN: Suppose we have a particle in one dimension, whose Lagrangian is

$$\mathcal{L} = -mc^2\sqrt{1 - \dot{x}^2/c^2} - V(x), \tag{2}$$

where m and c are positive constants. (Actually, m is the rest mass and c is the speed of light.) Assume $|\dot{x}| < c$.

(a). Find the Lagrange equation of motion for this particle.

(b). Find the canonical momentum $p = \partial\mathcal{L}/\partial\dot{x}$.

(c). Find the corresponding Hamiltonian $\mathcal{H}$.

(d). What is $\mathcal{H}$ in the limit $|\dot{x}| \ll c$?