Physics 827: Problem Set 8

Due Wednesday, November 21, 2007

1. Shankar, problem 10.3.2.
2. Shankar, problem 10.3.3.
3. Shankar, problem 10.3.6.
4. Prove the following properties of the angular momentum operators L_x, L_y, L_z, and $L^2 \equiv L_x^2 + L_y^2 + L_z^2$ defined in class:

   (a). $[L_z, L^2] = 0$.

   (b). $[L_z, L_\pm] = \pm\hbar L_\pm$ (where $L_\pm = L_x \pm iL_y$).

   (c). Consider $L_z = XP_y - YP_x$. In the coordinate representation, we would have
   $$L_z = x\left(-i\hbar\frac{\partial}{\partial y}\right) - y\left(-i\hbar\frac{\partial}{\partial x}\right). \qquad (1)$$
   Show that, in a two-dimensional system, if we make the coordinate transformation $x = \rho\cos\phi$; $y = \rho\sin\phi$, L_z takes the form
   $$L_z = -i\hbar\frac{\partial}{\partial\phi}. \qquad (2)$$

5. (20 pts.) (a). Show that $[L_z, P_x^2 + P_y^2 + P_z^2] = 0$.

   (b). Show that $[L_z, R] = 0$, where $R = \sqrt{X^2 + Y^2 + Z^2}$. Hint: use the identity $[P_x, f(X)] = -i\hbar f'(X)$, where $f(X)$ is a function of the operator X, and the prime denotes a derivative. Optional: prove this identity, assuming that $f(X)$ can be represented as a Taylor series.

   (c). Hence, show that $[L_z, V(R)] = 0$, where $V(R)$ is some function of R.

   (d). Show that $[L^2, V(R)] = 0$.

6. Shankar, exercise (12.3.8), parts (1) and (3) only. Note that this is a two-dimensional problem with a magnetic field perpendicular to the XY plane. This problem uses the fact that the Hamiltonian for a particle in a static magnetic field is $H = (\mathbf{p} - q\mathbf{A}/c)^2/(2m)$, where $\mathbf{p}$ is the canonical momentum and $\mathbf{A}$ is the vector potential. You do not have to prove this fact, but, if you are interested, you can read the proof in Shankar, pp. 90-91; I will also discuss it later in the year.