

## Physics 828: Problem Set VII

Dr. Stroud

Due Thursday, February 28 by 11:59 P. M.

Each problem is worth 10 pts. unless otherwise specified.

1. Shankar, problem 18.2.2.
2. Shankar, problem 18.2.6.
3. **Density of free particle states in one dimension.** In class, I showed that for a free particle of mass  $m$ , the density of states in energy  $\rho(E) = AE^{1/2}$ , where  $E$  is the energy, and  $A$  is a constant. We also derived the value of the constant. If the particle is spinless, we found that  $A = V(2m/\hbar^2)^{3/2}/(4\pi^2)$ , where  $V$  is the volume of the system, and  $\rho(E)$  is defined by the statement that the number of free particle states between  $E$  and  $E + dE$  is  $\rho(E)dE$ .

Now carry out the same calculation for a free particle of mass  $m$  in 1D. Again, as in class, use periodic boundary conditions to find the allowed values of  $k$  in a system of length  $L$  along the x axis. Show that the density of states in this case, assuming spinless particles, is

$$\rho(E) = \frac{L}{2\pi} \left( \frac{2m}{\hbar^2} \right)^{1/2} E^{-1/2}. \quad (1)$$

Note that this expression includes states of both positive and negative  $k$  of the same energy.

4. **Simple model of the photoelectric effect in one dimension.**

Consider, in a one-dimensional problem, a particle of mass  $m$  placed in a potential of the form  $V(x) = -\alpha\delta(x)$ , where  $\alpha$  is a real positive constant. Recall that in such a potential there is a single bound state, with energy  $E_0 = -m\alpha^2/(2\hbar^2)$ , corresponding to a normalized wave function  $\psi_0(x) = \sqrt{m\alpha/\hbar^2} \exp(-m\alpha|x|/\hbar^2)$ .

The particle (assumed to be of charge  $q$ ) interacts with an electric field oscillating with angular frequency  $\omega$ . The corresponding perturbation is  $W(t) = -q\mathcal{E}X \sin(\omega t) \equiv W \sin(\omega t)$ , where  $\mathcal{E}$  is a constant.

(a) Calculate the matrix element  $\langle k|X|\psi_0\rangle$  of the position observable  $X$  between the bound state  $|\psi_0\rangle$  and the positive energy state  $|k\rangle$  whose normalized wave function is approximated as a free particle state  $(1/L^{1/2}) \exp(ikx)$ .

(b). Hence, calculate, using the Fermi Golden Rule, the transition probability  $w$  per unit time to an arbitrary positive energy state. How does  $w$  vary with  $\omega$  and with the final energy  $E_f$  of the charged particle?

The Fermi Golden Rule, for this case, reads

$$w = \frac{\pi}{2\hbar} |k_f|W|\psi_0\rangle|^2 \rho(E_f = E_0 + \hbar\omega). \quad (2)$$

This differs by a factor of  $(1/4)$  from that given in class, because the time-dependence of the perturbation is  $\sin(\omega t)$  rather than  $\exp(-i\omega t)$ .

Comment: the final states, the density of final states, and the matrix element can actually all be calculated exactly.