

## Physics 828: Problem Set 2

Dr. Stroud

Due Wednesday, January 23 at 11:59:59 P. M.

Each problem is worth 10 points unless otherwise specified.

1. **Some properties of the Pauli spin matrices.**

(a). Verify Shankar, eq. (14.3.32).

(b). Verify Shankar, eq. (14.3.38).

(b). Verify Shankar, eq. (14.3.39).

2. Shankar, exercise 14.3.2.

3. Shankar, exercise 14.3.7.

4. (20 pts.) Consider a spin 1/2 particle. Call its spin  $\mathbf{S}$ , its orbital angular momentum  $\mathbf{L}$ , and its state vector  $|\psi\rangle$ . The two functions  $\psi_{\pm}(\mathbf{r})$  are defined by  $\psi_{\pm}(\mathbf{r}) = \langle \mathbf{r}, \pm | \psi \rangle$ . (These are the two components of the spinor wave function discussed in class.) Assume that

$$\begin{aligned}\psi_+(\mathbf{r}) &= R(r) \left[ Y_0^0(\theta, \phi) + \frac{1}{\sqrt{3}} Y_1^0(\theta, \phi) \right] \\ \psi_-(\mathbf{r}) &= \frac{R(r)}{\sqrt{3}} \left[ Y_1^1(\theta, \phi) - Y_1^0(\theta, \phi) \right]\end{aligned}\tag{1}$$

where  $r$ ,  $\theta$ , and  $\phi$  are the coordinates of the particle and  $R(r)$  is a given function of  $r$ .

(a). What condition must  $R(r)$  satisfy in order for  $|\psi\rangle$  to be normalized?

(b).  $S_z$  is measured with the particle in state  $|\psi\rangle$ . What results can be found and with what probabilities? Same question for  $L_z$  and then for  $S_x$ .

(c). A measurement of  $\mathbf{L}^2$  with the particle in state  $|\psi\rangle$  yielded zero. What state describes the particle just after this measurement?